III: OCEAN CIRCULATION

GLOBAL OCEAN DATA ASSIMILATION AND GEOID MEASUREMENTS

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Abstract. Parts of geodesy and physical oceanography are about to mature into a single modeling problem involving the simultaneous estimation of the marine geoid and the general circulation. Both fields will benefit. To this end, we present an ocean state estimation (data assimilation) framework which is designed to obtain a dynamically consistent picture of the changing ocean circulation by combining global ocean data sets of arbitrary type with a general circulation model (GCM). The impact of geoid measurements on such estimates of the ocean circulation are numerous. For the mean circulation, a precise geoid describes the reference frame for dynamical signals in altimetric sea surface height observations. For the time-varying ocean signal, changing geoid information might be a valuable new information about correcting the changing flow field on time scales from a few month to a year, but the quantitative utility of such information has not yet been demonstrated. For a consistent estimate, some knowledge of the prior error covariances of all data fields is required. The final result must be consistent with prior error estimates for the data. State estimation is thus one of the few quantitative consistency checks for new geoid measurements anticipated from forthcoming space missions. Practical quantitative methods will yield a best possible estimate of the dynamical sea surface which, when combined with satellite altimetric surfaces, will produce a best-estimate marine geoid. The anticipated accuracy and precision of such estimates raises some novel modeling error issues which have not conventionally been of concern (the Boussinesq approximation, self-attraction and loading). Model skill at very high frequencies is a major concern because of the need to de-alias the data obtained by the inevitable oceanic temporal undersampling dictated by realistic satellite orbit configurations.

1. Introduction

Physical oceanography and marine geodesy have historically had a long symbiotic history, including periods of discord. The most fundamental relationship has been through the shared problem of defining a marine geoid. For the geodesist, the geoid height is a fundamental quantitative description of the shape of the Earth. To the oceanographer it is a reference surface necessary for computing the oceanic circulation. Other branches of both sciences overlap, including the study of tides,

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"mean sealevel", Earth rotation and polar motion, and global and regional sealevel rise and fall.

Many of the issues which still confront us today were apparent in the debate between the late physical oceanographer R. B. Montgomery and the geodetic community about the apparent sealevel slopes along the US (and other regions) coastlines. The seemingly extremely accurate geodetic leveling surveys of that era produced for example, estimates that sealevel sloped upward to the north along the US east coast. Montgomery had pointed out that such a slope would be implausible because it implied that the Gulf Stream had to flow "uphill". However, physical oceanography at that time was widely regarded as an extremely crude subject, not capable of supporting comparison to the extremely accurate leveling surveys. Montgomery's student W. Sturges (Sturges, 1974) revived the debate later on and Montgomery was vindicated (Balazs and Douglas, 1974) when it was finally recognized that systematic errors were producing large-scale discrepancies in the leveling surveys.

The elements of the modern geodetic/oceanographic symbiosis are the same: the seasurface nearly, but not quite, coincides with the geoid; slopes of the seasurface relative to the geoid imply measurable oceanic velocities. Because the seasurface slopes of the seasurface are less than one meter in thousands of kilometers, small errors in estimates of the slopes imply large erroneous oceanic mass and property fluxes. Thus somewhat paradoxically, comparatively crude oceanic circulation estimates can provide relatively accurate estimates of the geoid height slopes. Modern attention to this problem arose with the development of high accuracy satellite altimetry and the various areas of overlap of physical oceanography and geodesy have led to a nearly complete convergence of issues. Here, we will focus primarily on the geoid/circulation problem.

Wunsch and Gaposchkin (1980) described the general problem and laid out the framework for combined estimation of the Earth's geoid and the ocean circulation. More than 20 years later, particularly with the flight of the TOPEX/POSEIDON altimetric spacecraft, enormous progress has been achieved. The problem is worth revisiting as we anticipate the flight of a new generation of spacecraft for determining the gravity field of the Earth by more direct methods (GRACE and GOCE; see Wahr et al., 1998; Drinkwater et al., 2002). In this note we discuss the status of the combined geoid/circulation estimation problem and elaborate on the evolving symbiotic relationship. Ideally, one should use a complete, joint, estimation procedure, but we lack the computational means to carry out the recipe in full. Ocean state estimation has, however, gained a degree of maturity that permits us today to obtain accurate estimates of the ocean circulation. This information can, and should, be used to improve understanding of the geoid.

Before launching into the substance of the problems, it is worth recalling just how far we have come: Consider Fig. 5 in Marsh and Chang (1978). One sees there two of the best marine geoid estimates, from circa 1977, along with the sealevel profile as measured by the GEOS-3 altimetric satellite. Discrepancies of 10 meters exist. Today, with all of the progress in satellite orbit and gravity field determination, discrepancies on these scales are well below one meter, and are in many places at the centimeter level (see the review by Tapley and Kim, 2001).

Here, we only briefly summarize the basic elements of the geoid/circulation problem. The fundamental relationship derives from the conclusion that the ocean is in near-hydrostatic equilibrium,

$$0 = -\frac{\partial p\,(\phi, \lambda, t)}{\partial z} - g\rho\,(\phi, \lambda, t) \tag{1}$$

where g is local gravity, z is the local vertical coordinate, ρ is the oceanic density, ϕ is latitude, λ longitude, and p is the pressure field. Knowledge of the horizontal gradient of the pressure field in the ocean is, in most places and times, sufficient to estimate the flow. At the seasurface, the gradients are easily shown to be

$$\begin{aligned} \frac{\partial p}{a\cos\phi\partial\lambda} &= \frac{g}{a\cos\phi}\frac{\partial\left[\zeta\,(\phi, \lambda, t) - N\,(\phi, \lambda)\right]}{\partial\lambda} \\ \frac{\partial p}{a\partial\phi} &= \frac{g}{a}\frac{\partial\left[\zeta\,(\phi, \lambda, t) - N\,(\phi, \lambda)\right]}{\partial\phi} \end{aligned} \tag{2}$$

where ζ is the surface elevation of the ocean relative to the center of the Earth, and N is the geoid elevation, here regarded as time-invariant (its expected time-variability is too small to affect these equations directly).

Although not entirely general, over the great volume of the ocean, the flow field at the seasurface is readily shown to be,

$$\begin{aligned} 2\Omega\sin\phi\rho v\,(\phi, \lambda, z = 0, t) &= \frac{g}{a\cos\phi}\frac{\partial\left[\zeta\,(\phi, \lambda, t) - N\,(\phi, \lambda)\right]}{\partial\lambda} \\ &\quad +\varepsilon_\lambda\,(\phi, \lambda, t) \qquad (3) \\ -2\Omega\sin\phi\rho u\,(\phi, \lambda, z = 0, t) &= \frac{g}{a}\frac{\partial\left[\zeta\,(\phi, \lambda, t) - N\,(\phi, \lambda)\right]}{\partial\phi} + \varepsilon_\phi\,(\phi, \lambda, t) \qquad (4) \end{aligned}$$

Ω is the Earth's rotation rate; u, v are the zonal and meridional velocity components. The terms $\varepsilon_{\phi,\lambda}$ are the errors from *both* sides of the equations that appear because the balance of terms is not perfect, and indeed an imbalance is an essential ingredient of the ocean circulation. The imbalances include non-linearity, time-dependence, and stresses. Nonetheless, from an observational point of view, attempts to directly measure the deviation from equality have generally failed to emerge from the noise level. The most obvious deviation from balance arises from the visual conclusion that the circulation evolves rapidly in time (see e.g., the oceanic animation at http://www.ecco-group.org). Despite this sometimes violent appearing variability on time scales of days and longer, the so-called geostrophic balance underlying (Eqs. 3, 4) remains extraordinarily accurate (with the notable exception of the ocean within about one degree of the equator).

From altimeters, the time-varying component of ζ is today known with overall accuracies approaching about 2 cm. The range of spatial variation of $\zeta - N$, in a long-term average, is no more than about 2 m (e.g., Wunsch and Stammer, 1998). If the ocean were at rest, one can make an estimate $\tilde{N}(\phi, \lambda) = \zeta(\phi, \lambda)$. Such a geoid would have errors of 2 m at most, and is more accurate than anything that was available 20 years ago; at high wavenumbers (beyond about spherical harmonic degree 20), it remains the most accurate available marine geoid. Alternatively, if N were known perfectly, then Eqs. (3, 4) would produce $u(\phi, \lambda, z = 0, t)$, $v(\phi, \lambda, z = 0, t)$, with an accuracy determined solely by the altimetric error in ζ, and far exceeding our actual present-day knowledge. Coupled with a knowledge of $\rho(\phi, \lambda, t)$, one would have sufficient knowledge to compute the full three dimensional time-evolving ocean circulation from Eqs. (2). In practice, neither (u, v), nor N is known perfectly and one seeks to estimate them jointly, both in the time mean and time-varying elements. The problem thus falls under the general subject of state estimation (or, in meteorological terminology, data assimilation).

2. The State Estimation Problem

In the most general terms, ocean state estimation aims to obtain the best possible description of the changing ocean and the external parameters governing its behavior by forcing the numerical model solutions to be consistent with all observations. This model-data combination, if carried out properly, results in a best-estimate ocean circulation – one that is better than can be obtained from either model or data alone. At the same time, the method also identifies model components that need improvement, including surface forcing fields, and produces guidelines to improved oceanic observing systems. (See Wunsch, 1996; Fukumori, 2001; Stammer et al., 2002a).

Because of the fundamental importance of understanding the present and future states of the ocean, the consortium "Estimation of the Circulation and Climate of the Ocean" (ECCO) was funded under the US National Ocean Partnership Program (NOPP) to obtain, through the application of mathematically rigorous assimilation methods, the best possible dynamically consistent estimates of the ocean circulation, which can serve as a basis for studies of elements important to climate (e.g., heat fluxes and variabilities). The ECCO consortium includes efforts at the Massachusetts Institute of Technology (MIT), the Jet Propulsion Laboratory (JPL), and the Scripps Institution of Oceanography (SIO). The resulting model-based syntheses and analyses of the large-scale ocean data set will enable a complete dynamical description of ocean circulation, including aspects that are not directly measured such as insights into the natures of climate-related ocean variability, major ocean transport pathways, heat and freshwater flux divergences (similar for tracer and oxygen, silica, nitrate), location and rate of ventilation, and of the ocean response to atmospheric variability.

Mathematically rigorous data assimilation is most commonly formulated as a least-squares problem in which an objective, or cost, function is minimized subject to, data and model dynamical constraints:

$$J = \sum_t (\mathbf{y}(t) - \mathbf{E}(t)\,\mathbf{x}(t))^T \mathbf{R}^{-1}(t)\,(\mathbf{y}(t) - \mathbf{E}(t)\,\mathbf{x}(t)), \tag{5}$$

where $\mathbf{y}(t)$ are observations distributed in space and time, $\mathbf{x}(t)$ is the model state, $\mathbf{E}(t)$ is an "observation matrix" that computes the model estimate of the observations – here assumed to be a linear combination of state vector elements. $\mathbf{R}(t)$ is the error covariance of the observations. $\mathbf{Q}(t)$ is the model-error covariance. The model, in the form,

$$\mathbf{x}(t+1) = \mathcal{F}[\mathbf{x}(t), \mathbf{q}(t), \mathbf{u}(t), \varepsilon(t), t], \tag{6}$$

is the (discrete-time) temporal evolution equation. Here, $\mathbf{q}$, $\mathbf{u}$ are the known and unknown, boundary conditions and problem parameters, respectively, and $\varepsilon(t)$ is the model error. It is assumed that $\langle \varepsilon(t)\,\varepsilon(t)^T \rangle = \mathbf{Q}(t)$, the model-error covariance, is known at least approximately. The model can be imposed upon the objective function J, either by using Lagrange multipliers, or in an "unconstrained optimization" form, using $\mathbf{Q}(t)$ as a weighting matrix in a penalty-function type of formulation in which J would be modified to,

$$J' = \sum_t \left[(\mathbf{y}(t) - \mathbf{E}(t)\,\mathbf{x}(t))^T \mathbf{R}^{-1}(t)\,(\mathbf{y}(t) - \mathbf{E}(t)\,\mathbf{x}(t)) + \varepsilon(t)^T\,\mathbf{Q}^{-1}\varepsilon(t) \right] \tag{7}$$

(This particular J' assumes that it makes sense to minimize the weighted sum of model and observational error; it is not the most general possibility.) The final solution is essentially a weighted least-squares fit of the model to the data with appropriate weights for both. Given the data and a model, the prescription of a priori errors associated with data and model constraints ($\mathbf{R}(t)$ and $\mathbf{Q}(t)$) dictates the quality of the assimilation product. The choice of weight matrices renders the solution, $\mathbf{x}(t)$, if it can be found, to be the maximum likelihood estimate for a linear model. As with all such estimation procedures, this one is reduced to a very large minimization problem.

ECCO state estimate computations are based on the MIT GCM (Marshall, et al., 1997); two parallel optimization efforts, the adjoint method (Lagrange multipliers or constrained optimization method) as described in Marotzke et al. (1999), and a reduced state Kalman filter smoother, e.g., Fukumori et al., (1999) are being used. First results of the global ECCO ocean state estimation based upon the method of Lagrange multipliers are summarized in Stammer et al. (2002a,b,c,d) and preliminary results from the sequential (filter/smoother) results are in Fukumori, et al. (1999). Data employed in ongoing synthesis calculations for the period 1992 through 2001 encompass the full WOCE data set and include absolute and time-varying altimetry, monthly mean sea-surface temperature data, WOCE hydrography, XBT, equatorial moorings (TAO-array) and profiling float (PALACE) temperature profiles, PALACE salinity profiles, mean surface drifter velocities, time-varying US National Center for Environmental Prediction (NCEP) re-analysis fluxes of momentum, heat, freshwater, and scatterometer wind stress fields. Monthly means of the model state are required to remain within assigned bounds of the monthly mean Levitus et al. (1994) climatology. In addition, and very important, the Lemoine et al. (1996) geoid estimate is used directly as the reference surface for the absolute altimetry, along with a full (non-diagonal) error covariance matrix; thus the best a priori existing geoid height estimate is employed along with a vast array of direct oceanographic observations. To bring the model into consistency with the observations, the initial potential temperature (θ) and salinity (S) fields are modified, internal viscosity and diffusion coefficients are estimated and the surface forcing fields are adjusted. Changes in those variables (often referred to as "control" terms) are determined through a best-fit (in the least-squares sense) of the model state to the noisy observations over the full data period. ECCO results are described, and can also be obtained, at http://www.ecco-group.org.

State estimates such as those being carried out in ECCO are computationally very demanding, involving the equivalent of iterative fitting of the model to the data over the entire data time-span. Sequential and Lagrange multiplier methods differ in the details of the computational overheads, but neither is trivial for models being run even at too-coarse resolution. On the other hand, the computations do represent the direction in which these fields must move: they are the only known ways to combine a complete knowledge of the dynamics with all of the data of any kind, and the efforts to render them more efficient and easy to use are going to continue apace.

A few central problems exist; two of them are related to the issue of errors. The first is the representation of the model error, written above as $\varepsilon(t)$. General circulation models are never accompanied by explicit statements of error, and of course, any given model will have different errors for different resolution and different elements, be it the mean temperature and salinity, or the low frequency wave propagation characteristics, or the structure of annual mean sealevel, among an infinity of other possible outputs. The degree of error, generally unknown therefore, controls the extent to which the state estimate fits the model relative to the data, and can make qualitatively important differences to the solution. Model errors are also of many different types, involving internal parameters of the model (mixing coefficients and the like), initial and boundary conditions, lack of resolution vertically and horizontally, in the specification of bottom topography and sidewall conditions, etc. Most existing model-error estimates are little more than guesses, and it is a high priority to learn how to represent model errors quantitatively. Some further discussion of some aspects of this problem is provided by Menemenlis and Chechelnitsky (1998).

Even with a fully specified model error, the state estimation error, i.e., the error of the estimate itself, involves the covariance structure of the full state. If the state vector contains N elements, in principle, at each time step, the second order statistics of the error alone are a matrix of dimension $N \times N$, and it evolves at each time step. Thus the error covariance structure of the mean seasurface involves propagation of model and data errors through a nonlinear system over many model years. The computational load is forbidding, and is behind much of the effort in the JPL-ECCO program to find useful approximations (see Fukumori, 2001). The absence of full error covariances remains the major limitation on the state estimation approach to geoid determination outlined here.

A successful determination of $\mathbf{x}(t)$ permits us to determine the oceanic flow field that is consistent with observations of all types, including purely geodetic ones and to make a best estimate of N that is also consistent with the observations and known oceanic dynamics. In the following, we will first discuss how improved GRACE and GOCE estimates of the geoid field will advance estimates of the ocean circulation. We will subsequently summarize how improved ocean estimates will feed back into the geoid estimation procedure.

3. Impact of the Geoid on State Estimates

It should be clear that improvements in the observations of *any* field leading to better estimates of $u\,(z = 0, x, y, t)$, $v\,(z = 0, x, y, t)$ will improve both the geoid, and the ocean circulation. Such observations include those of the geoid itself. New gravity field observations will improve the state estimation in various ways:

1. Improved geoid height fields reduce the error in Eqs. (2) and thus lead directly to improvements in oceanic circulation estimates. Fig. 1 shows the mean sea surface height field and a near-surface flow field, estimated from TOPEX/-POSEIDON data relative to the EGM96 geoid model (Lemoine et al., 1996), and which represents the current best-estimate absolute large-scale oceanic surface flow (see Stammer et al., 2002a).
2. Through the geoid height error covariance specification. All estimation problems have solutions whose quality is directly dependent upon the accuracy of the a priori error covariances.
3. Through new observations of bottom pressure at periods from 2 months to the mission lifetime, and which will provide information about the deep, time-varying ocean circulation that is otherwise generally completely unavailable. The degree to which this information represents qualitatively useful new constraints on the time-dependent circulation will only be known when the data are available.
4. Longer missions will be important for understanding secular trends in the ocean circulation. The ocean exhibits what are interpreted to be real trends, both globally (sealevel, temperature), and of opposing signs over large regions. Models also exhibit such trends, but it is difficult to separate real signals from model drifts owing to numerical approximations, and initial and boundary

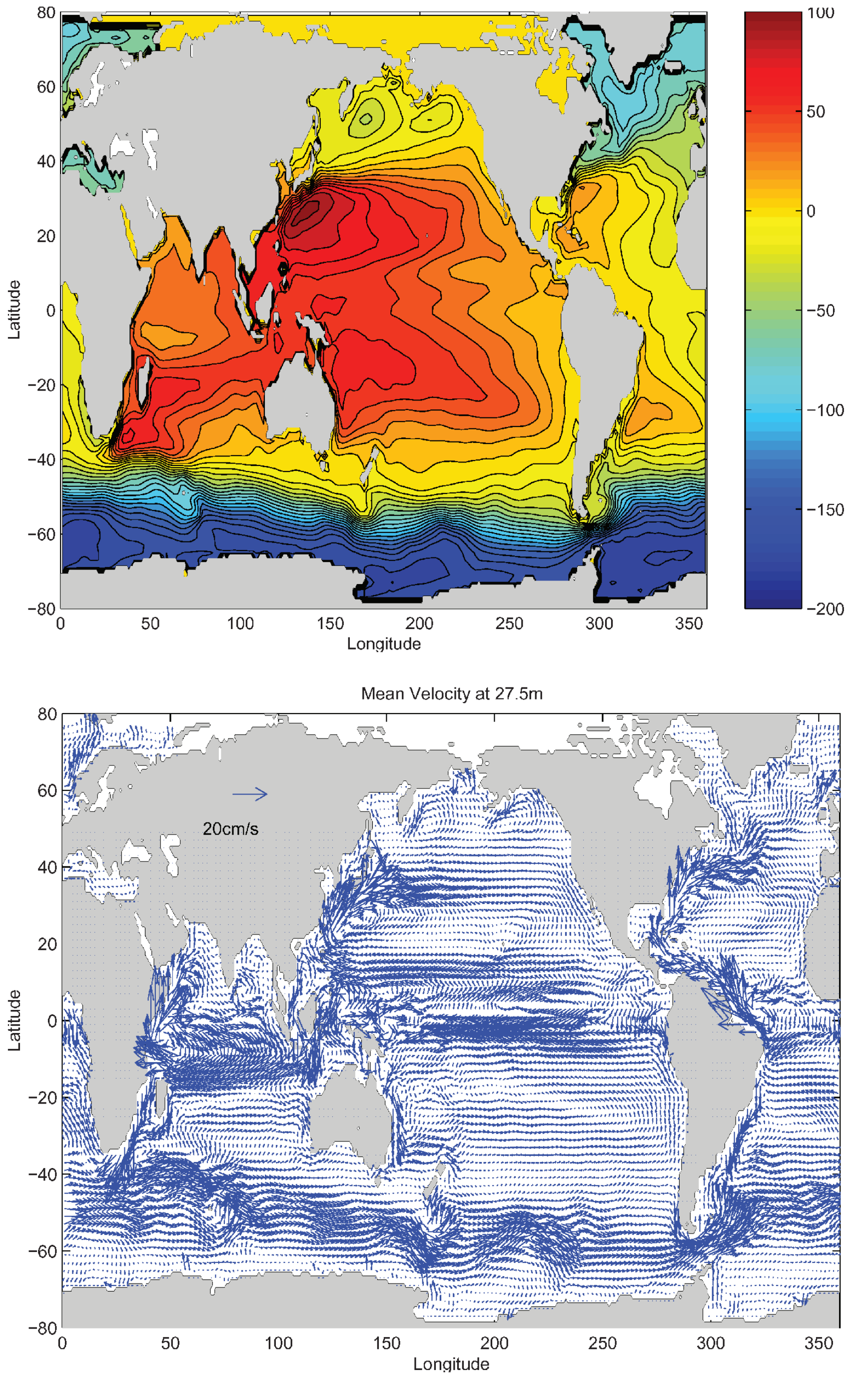

Figure 1. (top) Estimated mean sea surface height field (cm) as it results from the nine-year assimilation period is shown in the upper panel. (bottom) Mean estimated velocity field from 27.5 m (cm/s) from the same period (Stammer et al., 2002a).

condition errors. Gravity measurements should strongly constrain the mass redistribution trends in the ocean permitting separation of real signals from numerical artifacts.

4. Impact of Ocean State Estimates on Geodesy

As already noted, any estimate of oceanic surface flow is equivalent to a knowledge, along with the altimetric measurements, of the geoid height slope. There are several interactions between the state estimates and the geodetic inference problem.

1. Any ocean circulation estimate implies, along with an altimetric surface estimate, a geoid height estimate. Fig. 2 displays the best-estimate geoid from the latest ECCO results (Stammer et al., 2002a). It is visually indistinguishable from other geoids, and so the difference from EGM-96 is shown also. Such ocean-state-estimation geoids will improve as knowledge of the ocean circulation improves by whatever means is available (better theory, better models, better and additional data).
2. Conventional calibration of missions such as GRACE or GOCE is extremely difficult, if not impossible. What is possible are comparisons between GRACE-inferred fields and those independently determined. A major test of the absolute geoid determination from space is provided by the geoid estimate (Fig. 2) from information prior to the mission. A major obstacle here, and one for serious future work, is the great difficulty we have in providing formal uncertainties for the state estimates – the model is non-linear, and of very high dimension; the resulting computational load is currently beyond our capability of handling it.
3. We know (e.g., Stammer et al., 2000), that the gravity missions will alias the surprisingly energetic high frequency barotropic motions in the ocean, because its basic sampling interval is so long (nominally one month). Some of this energy would corrupt the mean state as well as the apparent time-varying geoid height. The best model estimates of that high frequency variability will come from the state estimates and these can be subtracted from the measurements. Model skill will be demonstrated by a measurable reduction in the variance of the resulting corrected fields relative to the uncorrected ones.
4. Oceanic (and atmospheric and core) motions affect the Earth's polar motion and rotation rate (see e.g., Ponte et al., 2001). These motions are of prime concern to geodesists as they affect the reference frames. As ocean models become more skillful, they will permit actual predictions of the polar motion (and as always, measurements of polar motion become useful constraints on the ocean circulation measurements in the general symbiosis).

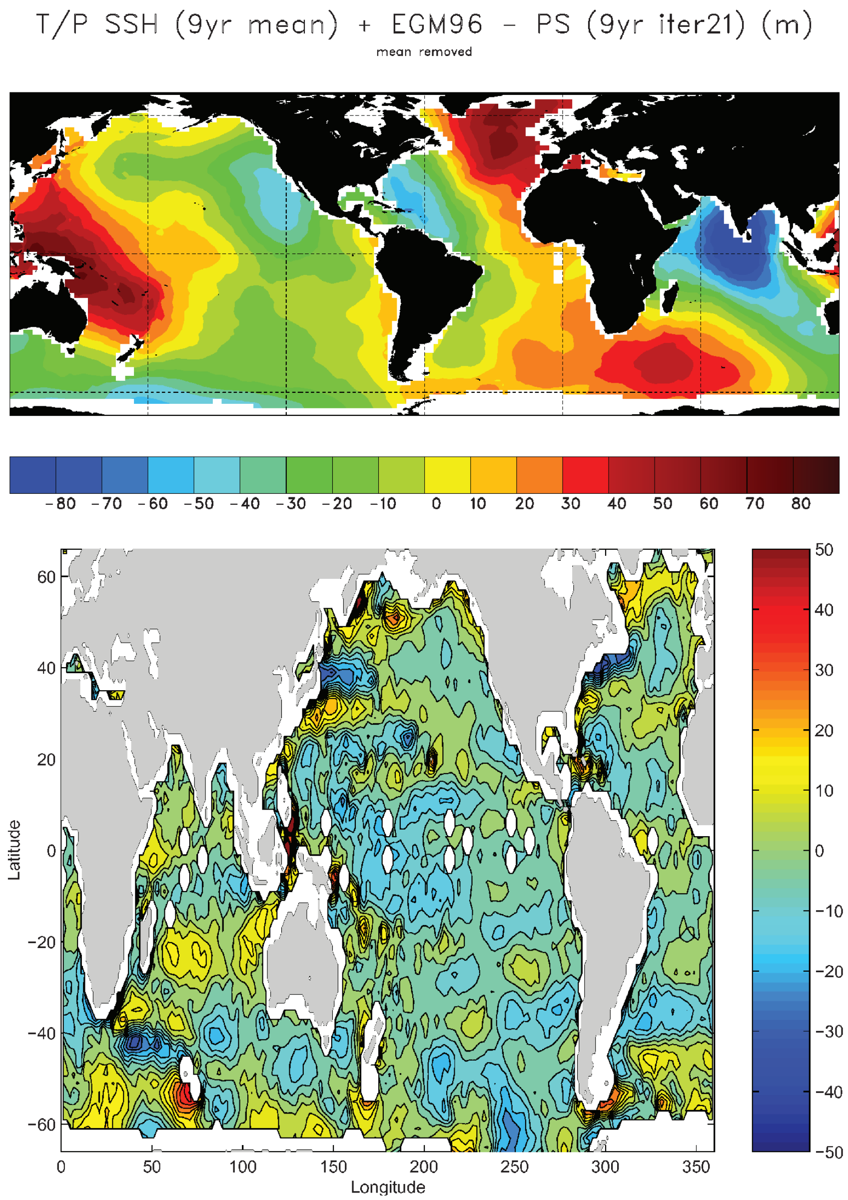

Figure 2. (top) Estimated mean geoid (in meters) as it results from subtracting Fig. 1a from a mean SSH field. (bottom) The estimated mean residual $\overline{\eta}_e - \overline{\eta}_{tp}$ in cm. Note data gaps in the tropical regions due to altimeter track pattern. All data over regions with water depth less than 1000 m were neglected here.

5. Skill

Without formal error estimates, determining the model skill has to be done through a series of comparisons between the model and data, both before and after the data are used as constraints. Note in particular, that the skill in Fig. 2 is tested directly against the in situ oceanic observations by the ability of the model to fit the entire suite of data. Any future geoid, produced by independent means, can be tested against the data by employing it with the model. Geoid height estimates will never be "validated"; rather they will either provide oceanic model estimates consistent with the in situ data, or not so-consistent. (For a useful discussion of the fallacy of "validation", see Oreskes, et al., 1994.)

As an example of the direct testing of the model against data, we show in Fig. 3 a comparison of two comparatively long records from the Southern Ocean (Spencer et al., 1993) with results at that location from both the unconstrained and constrained models. The unconstrained model clearly tracks the bottom pressure reasonably well. The constrained model does somewhat better, particularly at low frequencies.

A more quantitative comparison of the three estimates for each gauge can be seen in Fig. 4. The spectra of model and data are quite similar at all frequencies. In this preliminary result from the constrained model, most of the increased skill is at low frequencies (the coherence increases in the constrained model there). Note however, that errors of 1 cm of water can create very large oceanic transport errors if they occur over finite distances. This sensitivity is what leads to the expectation that time dependent ocean bottom pressure variations may be powerful constraints on models, and indirectly influence the geoid height estimate.

6. Modeling Issues

Apart from the difficulties mentioned with the absence of model error estimates, the realistic possibility for oceanic bottom pressure measurements from space with precisions approaching or exceeding 1mm of water equivalent (see for example, Wahr et al., 1998; Drinkwater et al., 2002), raises a number of challenging issues for modeling. Hitherto, the need to model, and to use data of this precision, has not been an issue as the data were either non-existent, or much coarser than now anticipated. At the level of precision of GRACE, some conventional approximations used in almost all models begin to fail. These include the so-called Boussinesq approximation that treats the fluid as essentially incompressible (see e.g., McDougall et al., 2002; deSzoeke and Samelson, 2002, and the references there). A perhaps more surprising issue is that discussed by Dewar et al. (1998), who show that the approximation of using depth as a surrogate for pressure in the equation of state of seawater leads to fictitious abyssal pressure gradients of several centimeters of water. Undoubtedly other approximations will have to be dealt with. One example

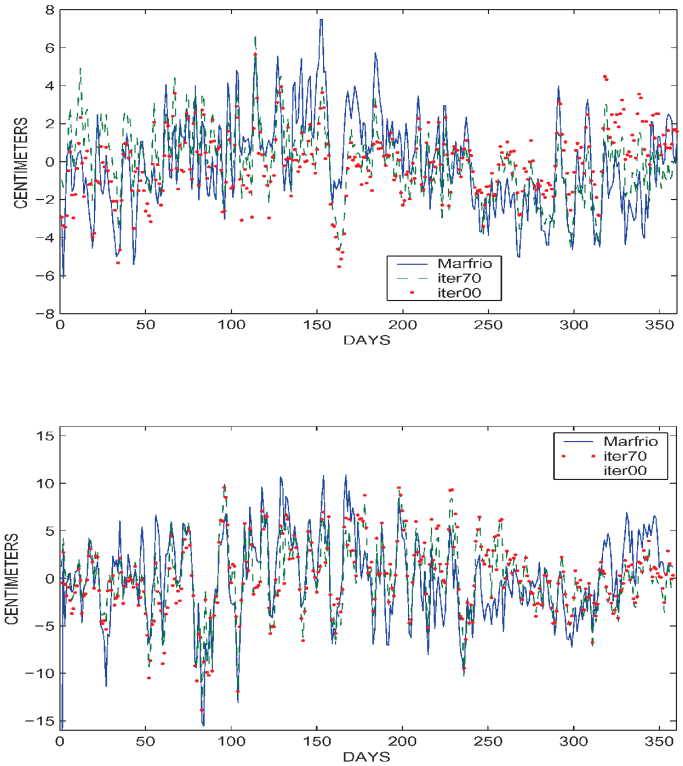

Figure 3. Comparison of ocean bottom pressure variations with an unconstrained model (labelled iter 00), with a constrained one (after 70 iterations). Upper panel is for an instrument at 31°60′S 36°00′W, 2604 m depth, and lower panel is for an instrument at 46°52′S, 52°28′E in 3600 m of water.

is the difficulty in specifying topography in models: the seafloor contains all spatial scales and it is geographically quite inhomogeneous. Typically, topography is averaged over some fixed distance; whether such averages are adequate parameterizations of all of the sub-grid scale topographic effects (scattering) is doubtful. The closure of passages that should be open, by averaging, affects abyssal water mass properties, and hence the gravity field distribution. In many regions, the real topography remains inadequately sampled (further discussion of some elements of the general topographic problem can be found in Losch and Wunsch, 2003).

A somewhat novel issue concerns the extent to which oceanic loads and self-attraction generate measurable effects on the gravity field seen from space. Although these problems are well-known in ocean tidal modelling, they are new in the general circulation context, and are taken up by Condi and Wunsch (2002).

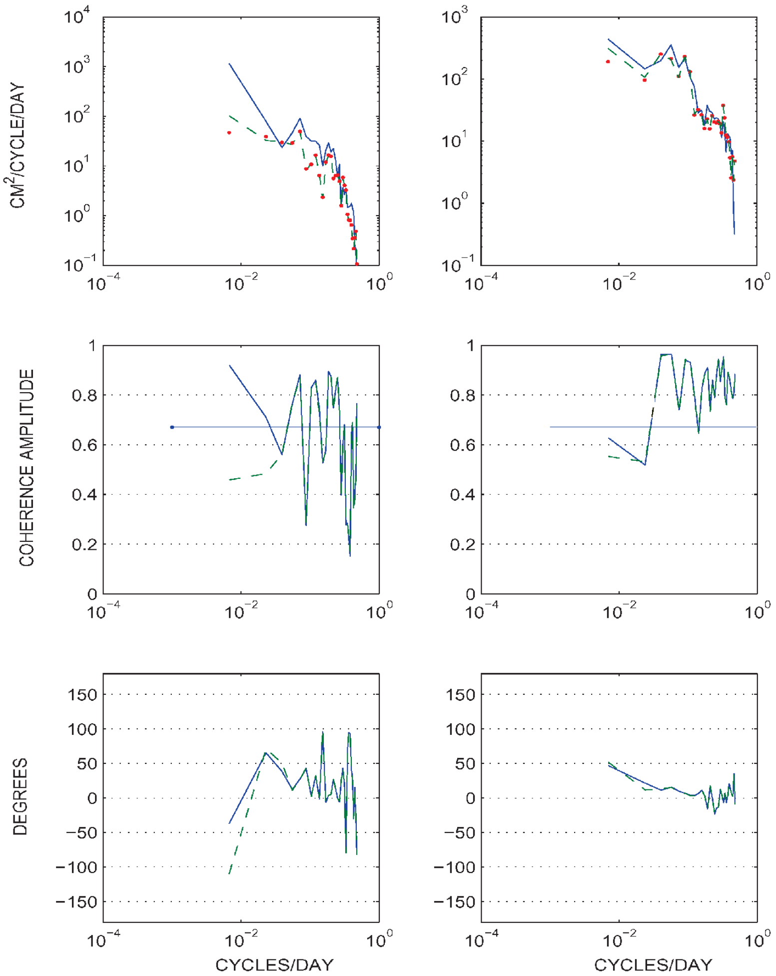

Figure 4. Spectra (upper panels) from the two gauges and the two model estimates shown in 3. Middle panel is the coherence of the records with the two model runs (amplitude) and the lower panels are the coherence phases. An approximate level of no significance for the amplitude is shown.

7. Outlook

The convergence of many aspects of physical oceanography and geodesy can be expected to continue and eventually will mature into a single modeling problem, that will greatly advance both fields. Any information, observational or theoretical, that improves the marine geoid estimates leads to better estimates of the ocean circulation and vice-versa. The most general machinery we have available for using any information of almost any character is that of state estimation in which the data are combined with dynamical and kinematical requirements. Oceanic state estimation has evolved to the stage where both absolute and temporally varying geoid height information can be combined with oceanic observations, either from space, or in situ, to produce simultaneous best-estimates of both the marine geoid and the ocean circulation. We anticipate continued improvements in the methodologies and as data from new space gravity-measuring missions become available, the community should be able to employ the data nearly routinely. The chief computational problem at the present time is the computational load involved in finding the formal uncertainty estimates for the combined fields; the major conceptual issue is the difficulty in specifying model errors.

A somewhat different role for oceanic state estimates is in the required computation of oceanic time-varying motions at high frequencies so as to reduce the aliasing of missions that necessarily undersample the time-variable ocean. These motions include the tides (which we have not focussed on here), but also the stochastic continuum motions that are most conspicuous at high latitudes. The skill of these models, and their ability to reduce the observed variance, can also be expected to improve as a data stream emerges (Recall Fig. 3.)

Of the two missions specifically dealt with here, GRACE and GOCE, the former provides the novel prospect of observing time-varying ocean bottom pressures, and the latter is expected to provide accurate time average geoids to higher spherical harmonic degree and order than will GRACE. At our present state of experience, it is not clear which, if either, will prove to be the more powerful in setting constraints to improve estimates of either the ocean circulation or geoid height or both.

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References

Balazs, E. I. and Douglas, B. C.: 1979, 'Geodetic leveling and the sealevel slope along the California coast', *J. Geophys. Res.* **84**, 6195–6206.

Condi, F. and Wunsch, C.: 2002, 'Measuring gravity field variability, the geoid, ocean bottom pressure fluctuations, and their dynamical implications', *J. Geoph. Res.*, submitted for publication.

Dewar, W. K., Hsueh, Y., McDougall, T. J., and Yuan, D.: 1998, 'Calculation of pressure in ocean simulations', *J. Phys. Oceanogr.* **28**, 577–588.

Drinkwater, M. R., Floberghagen, R., Haagmans, R., Muzi, D., and Popescu, A.: 2002, 'GOCE: ESA's First Earth Explorer Core Mission', *Space Sci. Rev.*, this volume.

Fukumori, I.: 2001, 'Data assimilation by models', in *Satellite Altimetry and Earth Sciences*, L.-L. Fu and A. Cazenave (eds.), Academic, San Diego, 237–266.

Fukumori, I., Raghunath, R., Fu, L.-L., and Chao, Y.: 1999, 'Assimilation of TOPEX/POSEIDON altimeter data into a global ocean circulation model: How good are the results?', *J. Geophys. Res.* **104**, 25,647–25,665.

Lemoine, F., et al.: 1997, 'The development of the NASA GSFC and NIMA Joint Geopotential Model', in Segawa et al. (eds.), *Proceedings of the International Symposium on Gravity, Geoid and Marine Geodesy,* International Association of Geodesy Symposia, Vol. 117, 461–469, Springer-Verlag, Berlin.

Levitus, S., Burgett, R., and Boyer, T.: 1994, *World Ocean Atlas 1994*, vol. 3, *Salinity*, and vol. 4, *Temperature*, *NOAA Atlas NESDIS 3 & 4*, U.S. Dep. of Comm., Washington, D.C.

Losch, M. and Wunsch, C.: 2003, 'Bottom topography as a control variable in an ocean model', *J. Atmos. and Ocean Techn.*, in press.

Marsh, J.G, and Chang, E.S.: 1978, '5' detailed gravimetric geoid in the northwestern Atlantic Ocean', *J. Mar. Geodesy* **1**, 253–261.

McDougall, T. J., Greatbatch, R. J., and Lu Y.: 2002, 'On conservation equations in oceanography: how accurate are Boussinesq models?', *J. Phys. Oceanogr.* **32**, 1574–1584.

Marotzke, J., Giering, R., Zhang, Q. K., Stammer, D., Hill, C. N., and Lee, T.: 1999, 'Construction of the adjoint MIT ocean general circulation model and application to Atlantic heat transport sensitivity', *J. Geophys. Research* **104**, 29,529–29,548.

Marshall, J., Adcroft, A., Hill, C., Perelman, L., and Heisey,C.: 1997, 'A finite-volume, incompressible navier-stokes model for studies of the ocean on parallel computers', *J. Geophys. Res.* **102**, 5753–5766.

Menemenlis, D. and Chechelnitsky, M.: 1998, 'Error estimates for an ocean general circulation model from altimeter and acoustic tomography data', *Monthly Wea. Rev.* **128**, 763–778.

Oreskes, N., Shrader-Frechette, K., and Belitz, K.: 1994, 'Verification, validation, and confirmation of numerical models in the earth sciences', *Science* **263**, 641–646.

Ponte, R. M., Stammer, D., and Wunsch, C.: 2001, 'Improved ocean angular momentum estimates using an ocean model constrained by large-scale data', *Geophys. Res. Letters* **28**, 1775–1778.

Spencer, R., Foden, P. R., McGarry, C., Harrison, A. J., Vassie, J. M., Baker, T. F., Smithson, M. J., Harangozo, S. A., and Woodworth, P. L.: 1993, 'The ACCLAIM program in the South Atlantic and Southern Oceans', *Intl. Hydrog. Rev.* **70**, 7–21.

Stammer, D., Wunsch, C., and Ponte, R.: 2000, 'De-Aliasing of global high frequency barotropic motions in altimeter observations', *Geophysical Res. Letters* **27**, 1175–1178.

Stammer, D., Wunsch, C., Giering, R., Eckert, C., Heimbach, P., Marotzke, J., Adcroft, A., Hill, C.N., and Marshall, J.: 2002a, 'The global ocean circulation during 1992–1997, estimated from ocean observations and a general circulation model', *J. Geophys. Res.*, 107(C9), 3118, DOI: 10.1029/2001JC000888.

Stammer, D., Wunsch, C., Fukumori I., and Marshall, J.: 2002b, 'State estimation in modern oceanographic research', *EOS, Transactions, American Geophysical Union*, 83(27), 289&294-295.

Stammer, D., Wunsch, C., Giering, R., Eckert, C., Heimbach, P., Marotzke, J., Adcroft, A., Hill, C.N., and Marshall, J.: 2002c, 'Volume, heat and freshwater transports of the global ocean circulation 1992–2000, estimated from a general circulation model constrained by WOCE data', *J. Geophys. Res.,* 108 (C1), 3007, DOI: 10.1029/2001JC001115.

Stammer, D., Ueyoshi, K., Large, W.B., Josey S., and Wunsch, C.: 2002d, 'Improving air-sea flux estimates through global ocean data assimilation', *ECCO Report Series, Report 13, pp: 31 (see also http://www.ecco-group.org/reports.html).*

Sturges, W.: 1974, 'Sea level slope along continental boundaries', *J. Geophys. Res.* **79**, 825–830.

de Szoeke, R. A. and Samelson, R. M.: 2002, 'The duality between the Boussinesq and non-Boussinesq hydrostatic equations of motion', *J. Phys. Oc.* **32**, 2194–2203.

Tapley, B. D. and Kim, M.-C.: 2001, 'Applications to geodesy', in *Satellite Altimetry and Earth Sciences*, L.-L. Fu and A. Cazenave (eds.), Academic, San Diego, 371–406.

Wahr, J., Molenaar, M., and Bryan F.: 1998, 'Time variability of the Earth's gravity field: hydrological and oceanic effects and their possible detection using GRACE', *J. Geophys. Res.* **103**, 30,205–30,229.

Wunsch, C.: 1996, *The Ocean Circulation Inverse Problem*, 442 pp., Cambridge Univ. Press, New York.

Wunsch, C. and Gaposchkin, E. M.: 1980, 'On using satellite altimetry to determine the general circulation of the oceans with application to geoid improvement', *Revs. Geophys and Space Phys* **18**, 725–745.

Wunsch, C. and Stammer, D.: 1998, 'Satellite altimetry, the marine geoid and the oceanic general circulation', *Ann. Revs. Earth Plan. Scis.* **26**, 219–254.

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