Extremes, Patterns, and Other Structures in Oceanographic and Climate Records

DRAFT

Carl Wunsch

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"My eye is better than any statistical test." Well-known paleoceanographer, circa 2001.

1 Introduction

Many of the most important inferences that are made about the climate system and its changes are based upon statistical or probabilistic inferences. These inferences include the deduction of occurrence of extreme events in a general sense (including e.g., exceptionally large El Niño patterns, stronger than expected trends, covariances beyond that expected between disparate records, unusual "runs"). Probability and statistics are, however, among the least intuitively accessible of all mental constructs, and a very large literature now exists (e.g., Kahneman et al., 1982; Gilovich et al., 2002) showing how counter-intuitive many important statistical inferences are with enormous implications not only for science, but for economics and public policy generally. Kahneman, although a psychologist, won the Nobel Prize in economics because his discussions of how people make decisions in the presence of uncertainty captured so much of their real (as opposed to ideal) behavior. A classic discussion of economic behavior is Mackay (1852); a more recent one is Malkiel (1999). Coupled with a very powerful human instinct that the world must be deterministic (predictable)[1], the scientific literature too, is riddled with misguided or mistaken conclusions. A number of authors (e.g., Diaconis and Mosteller, 1989) have noted the very great difficulties people (including scientists) have in dealing with apparently amazing, but expectable, coincidences.

[1] "Human nature likes order; people find it hard to accept the notion of randomness. No matter what the laws of chance tell us, we search for patterns among random events wherever they might occur..." (Malkiel, 1999).

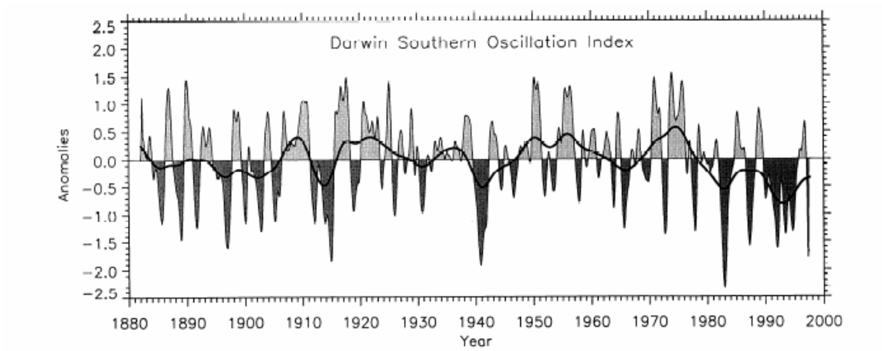

Figure 1: Negative of the anomaly of atmospheric surface pressure at Darwin, Australia (Trenberth and Hoar, 1997) and used by them as a measure of the Walker circulation strength. {trenberth&hoa

This paper has a primarily pedagogical intention—there are no original results; rather it arises out of attempts to teach students some of the most basic of statistical inference skills, while simultaneously cautioning them to remain open-minded and and skeptical.

Most of what follows is nothing more than a series of examples where some statistical or probabilistic inference was made that is less clear-cut than the authors' seemed to believe, and/-or where a certain skepticism would have been better retained despite the temptation to make an exciting inference. The possible connection with the widespread wish for attention in the popular media will be obvious. An earlier more limited version of these examples was discussed by Wunsch (1999).

2 Single Time Series Examples

2.1 Time Domain

Consider Fig. 1 showing the negative of the anomaly of atmospheric pressure in Darwin, Australia and used by Trenberth and Hoar (1997) as a measure of the strength of the Walker circulation. Fig. 2 displays the sea surface temperature (SST) Niño3.4 index commonly used as a measure of the strength of El Niño. Trenberth and Hoar (1997) infer that ENSO behavior shifted after about 1970 to more frequent and larger El Niño events (The change was interpreted as the result of global warming and to be "unprecedented" in the historical record.) Solow (2006), however, noted that their anomalous test period, 1992-mid-1995, in Fig. 1 was not independent of the earlier supposedly normal behavior; he used the subsequently longer record to recalculate the probability that the nature of ENSO had changed in some significant way. His conclusion was, in contrast, that while the test period appears different from the earlier one,

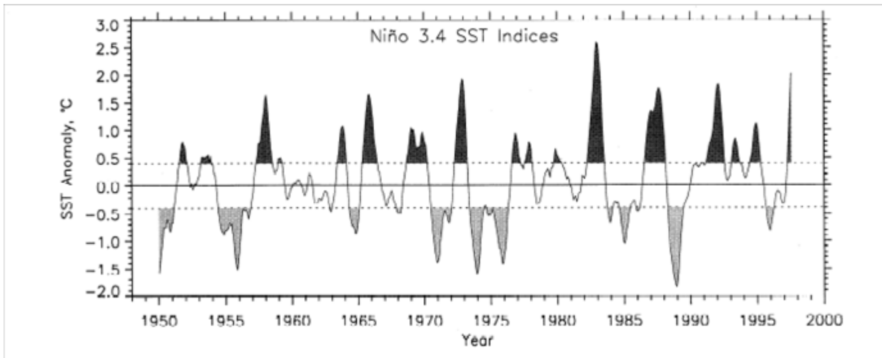

Figure 2: Time series of the Niño-3 index (from Trenberth and Hoar, 1997) {trenberth&hoa

subsequently there was essentially no evidence that the nature of the physics had changed. The comparatively dramatic story of the original authors is thus replaced by a much more ambiguous and unexciting, but presumably more soundly-based, description of the nature of ENSO.

Another example of simple pitfalls can be seen in Fig. 3. Hurrell and van Loon (1997) inferred that the apparent trend at the end of this record represented another unprecedented shift in a climatologically important index that of the so-called North Atlantic Oscillation (NAO). Wunsch (1999) showed, however, that such *apparent* trends were characteristic of ordinary red noise processes and *did not necessarily* have any significance beyond that of a random walk. Subsequently, Percival and Rothrock (2005) noted that the statistical significance of such a trend, if calculated using ordinary least-squares fits over the interval from 1965 to the record end, could preduce results differing by an order of magnitude from the correct inference. The gist of their argument was that the conventional tests do not permit one to choose the interval for the test by visual examination—that has the effect of using posterior knowledge (the inference of a region with an apparent trend) with a statistical test that assumes no such knowledge. These debates are related to those about "regime changes" (Rudnick and Davis, 2003; Overland et al., 2006 and subsequent comments) and non-stationarity. For the latter, consider Fig. 4 showing a record that many would perhaps visually declare to be non-stationary, the major event near time 1300 appearing out of character with the previous and later intervals. This record is in fact stationary, having been generated as the cube of an AR(1) process (Wunsch, 1999). Visual inspection is quite misleading, and one would need to attempt to distinguish the hypothesis of non-stationarity from non-normal behavior of an autocorrelated process—a very difficult problem with small sample sizes.

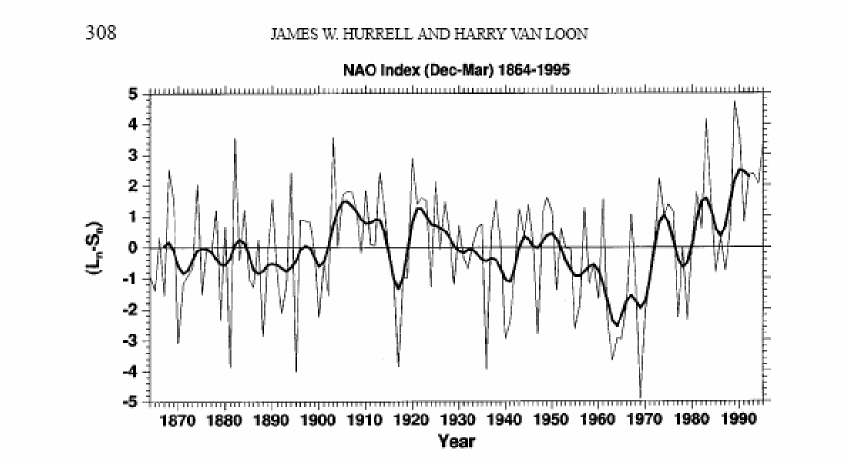

Figure 3: Thin line shows the so-called winter North Atlantic Oscillation (NAO) Index from Hurrell and van Loon (1997). Thick line is a low-pass filtered version. The subject of discussion is the apparent trend beginning about 1960, and inferred to represent a climate change. {hurrell&vanlo

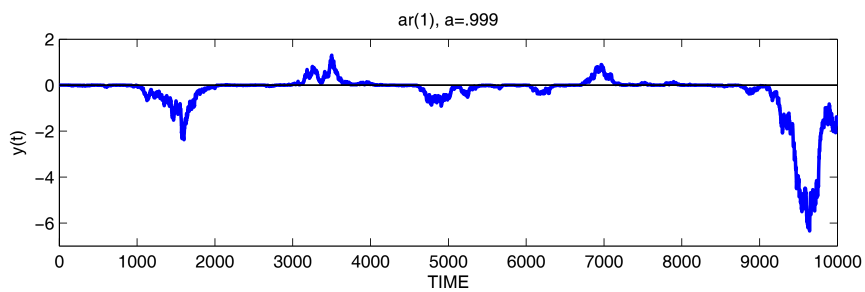

Figure 4: Example of a record that might be inferred by visual inspection to have non-stationary behavior. In practice, it is simply the cube of a stationary AR(1) process, $x\left(n\right)=0.999x\left(n-1\right)+\theta\left(n-1\right),$ where $\theta\left(n\right)$ are iid Gaussian pseudo-random. Small sample determination of non-normal (which this is) as distinguished from non-stationary behavior is difficult. {cubedar_1.eps

3 How Significant Must a Signal Be?

For most of the history of physical oceanography and climate studies generally, the results were primarily of interest world-wide to a handful of mainly academic scientists. The stakes, from the point of view of the wider community, were low, and no dire effects arose from mistaken inferences. A commonly accepted measure of significance was beyond one standard deviation (a 65% confidence interval for Gaussian distributions), or 95% confidence intervals or levels for Fourier analyses. One might contrast that situation with e.g., a drug trial, where one would like to be confident at the 99.9% level, that a new drug does not generate excess mortality in its users. In a world troubled by climate shifts, and the tendency of the latest scientific results to make headline news, the impact of erroneous inferences in climate and oceanography now can have enormous social consequences (see e.g., Kerr, 2006). Much of the comfortable scientific obscurity has vanished.

A level-of-significance of 95% implies the expectation of about 5% false positives. With one million independent samples, there will be 50,000 false signals (large sample fluctuation extremes become an issue e.g., in David Thomson's tests for true periodicities in geophysical records; see Percival and Walden, 1993, P. 513, who quote him as suggesting confidence levels above $1 - 1/N$ where N is the number of samples.) In an interesting paper, Seife (2000) discusses numerous physics signals apparently signficant at the five and six standard deviation level that ultimately proved ephemeral or simply incorrectly interepreted. The astronomer John Bahcall is quoted as observing "Half of all three-sigma results are wrong."

In contrast, and in a different but related, problem worth mentioning, Lanzante (2005) makes note of the tendency to compare overlaps of ordinary error bars as a test of consistency of data sets. He notes that such a comparison can be quite misleading—and overly conservative. Where feasible, one needs to use two-sample rather than one-sample tests.

3.1 Fourier Methods

Spectra

Fourier spectral density estimates are very powerful analysis tools in many contexts. As a combination, however, of Fourier analysis and statistics, they are perhaps unrivaled in the literature in giving rise to elementary misconceptions. Consider for example, the spectral densities displayed in Fig. 5 for the purpose of conveying the importance of the so-called Milankovitch cycles in ice core records. The peaks (here considered "extreme events") strongly suggest record dominance by the astronomical forcing periods lying in the vicinity of 100, 41 and 21 thousand years. (Notice the absence of any uncertainty estimates inthese figures).

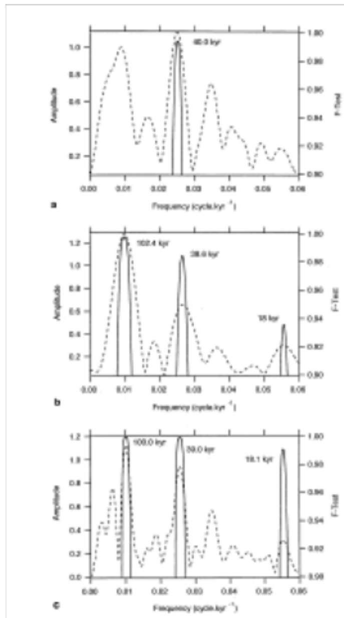

Figure 5: From Vimieux et al. (2001) showing spectral peaks in a proxy measured in an Antarctic ice core and analyzed over different time intervals. Note the linear power scale and the small frequency interval displayed.

{Vimieux1}

If one plots the power density spectral estimate over the entire frequency band, not just that restricted part visible in Fig. 5, and uses a logarithmic power scale (so that the displayed 95% confidence interval is constant with power), one infers a completely different picture of the underlying time series: one in which the record is described at zero-order as nearly red-noise, and in which the apparent peaks at the astronomical frequencies, while likely real, contain only a very small fraction of the record variance (see Wunsch, 2004). Hundreds of examples of such exaggerated display of "peaks" can be found in the literature. Often the energy fraction in the peak is so slight as to be of truly questionable importance. (For example, see Wagner et al., 2001, for the so-called solar DeVries cycle.)

Bivariate and Multivariate Correlation and Coherence

Perhaps the most insidious issues lie with inferences that two or more records are correlated. Indeed, much of the paleoclimate literature indulges in what is candidly labelled "wiggle matching". In that method, two spatially distant records with uncertain time bases are assumed to be related, the time axes being then shifted to align the events in the records. Sometimes after shifting, correlations are calculated and proclaimed to be statistically significant (e.g., Bond et al., 1991). Fig. 6 displays a classical example of records apparently showing compelling visual correlation, but that disappeared when the record became long enough (see Pittock, 1978)

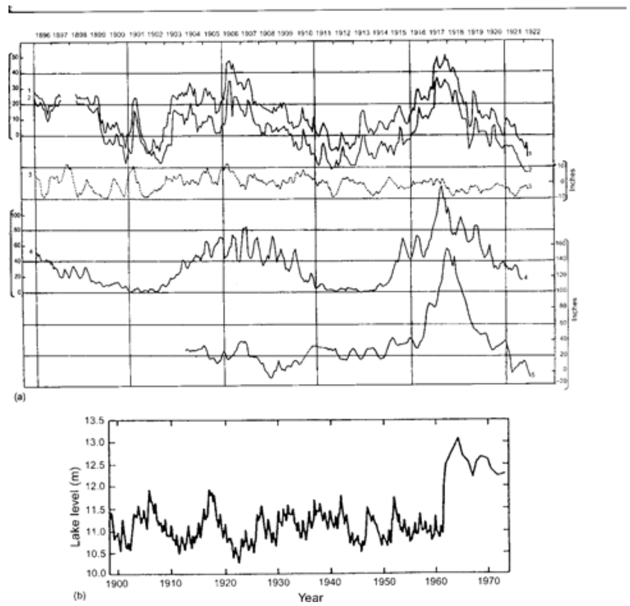

Figure 6: Upper panel is from Brooks (1923) showing the apparent correlation of Wolf sunspot numbers with central African lake levels. Lower panel shows the disappearance of the apparent sunspot cycle in the lake levels (reproduced from Pittock, 1978). {brooks_pittoc

The possibility of excursions of any stationary Gaussian time series far from the apparent mean value can be considered in several ways, including the use of extreme value statistics. Alternatively, the approach of Rice (1945), summarized e.g. by Cartwright and Longuet-Higgins (1956) or Vanmarcke (1983) provides considerable insight. In that approach, it is shown that for any time series with a known power spectral density $\Phi(\omega)$, that the rate of threshold crossings (upward or downward) depends only upon the low order spectral moments,

$$\lambda_n = \int_0^{\pi/2\Delta t} \omega^n \Phi(\omega)\, d\omega,$$

where Δt is the sampling interval. Unless one can show that some particular excursion is exceptional, no claim can be made that anything unusual has transpired.

In other examples (e.g., Chapman and Shackleton, 1999), coherences between records are computed, sometimes properly, but at such low levels of no significance (in that example, 80%) that the number of frequency bands apparently related in the two records is so large as to vitiate the entire test. Another example, shown in Fig. 7 was clearly intended to convince the reader by visual inspection that the two records were causally related,.and that the time-lag of six years introduced in the time shift was justified by the crosscorrelation also displayed. It is left to the reader to judge whether these two records are convincingly connected. A few more examples

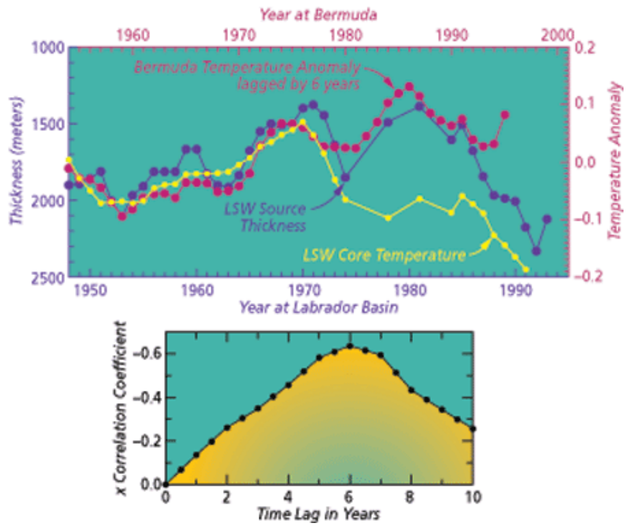

Figure 7: Curry and McCartney (1996) in which the red and blue curves are inferred to be causally related. Note that the red curve has been shifted by six years relative to the blue one. The number of statistical degrees of freedom here is presumably very small.

{curry_mccartn

are discussed by Wunsch (2006).

A common practice in the geophysical sciences is to combine theory with empiricism and to relax even the usual weak tests of statistical signficicance. One example in which I was directly involved concerns the so-called pole tide. The Earth's rotation axis wobbles about the geographical north pole with a period of approximately 14 months (see Munk and Macdonald, 1960). This motion induces a changing centrifigual force at all locations, and which would act dynamically as a broadband tide.

Analysis of periodograms and spectral density estimates of long tide gauge records showed (see Fig. 8) excess energy in this band only in the North Sea region, increasing eastward into the Baltic Sea. The apparent amplitude was several times that expected for so-called "equilibrium tide". A formal analytical theory explaining this phenomenon was offered (Wunsch, 1986). Subsequently, attempts to reproduce the phenomenon with numerical models proved negative; furthermore, as the record length subsequently grew, the apparent pole tide strongly diminished with time. It now appears (Wunsch, 2001 and the references there) that the signal was nothing but the random fluctuation of energy in the meteorological forcing, coincidentally in the pole tide frequency band, and that the entire oceanographic literature on this subject was directed at a will-o'-the-wisp. As this topic has been of interest to a tiny scientific community, no great harm was done. It does strongly suggest, however, that one should use more formal Bayesian methods (e.g., Gauch, 2003) in combining a priori theoretical knowledge with observations (although

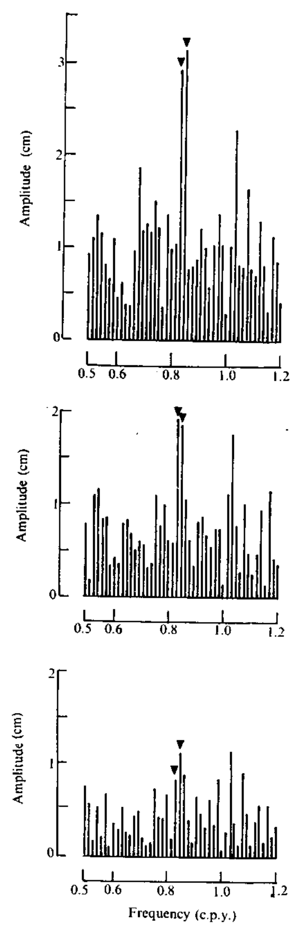

Figure 8: From Miller and Wunsch (1973) showing the growth of periodogram amplitudes with distance into the North Sea. The period of excess energy coincides with that known from the Chandler Wobble and was inferred to represent a non-equilibrium "pole tide". With hindsight, the simplest explanation is just a random fluctuation in the periodogram. {miller_wunsch

Planet	i	Semi-Major Axis (AU)	Titius-Bode Law (AU)
Mercury	-∞	0.39	0.4
Venus	0	0.72	0.7
Earth	1	1	1
Mars	2	1.52	1.6
Ceres	3	2.77	2.8
Jupiter	4	5.2	5.2
Saturn	5	9.54	10
Uranus	6	19.18	19.6
Neptune	7	30.06	38.8
Pluto	8	39.44	77.2

Figure 9: The Titius-Bode Law of planetary separation in AU (Murray and Dermott, 1999). Is the rule coincidence?

{titius_bodela

precisely what was prior and what was posterior in such situations is not completely obvious).

4 Patterns and Formulas

The so-called Titius-Bode Law, providing a formula for the spacing of the planets (including Pluto), has been known for hundreds of years. It says, in one form, that the spacing (in astronomical units) is $d = 0.4 + 0.3 \times 2^j$ where j is the j-th planet (see Fig. 9 and Murray and Dermott, 1999). Work over hundreds of years has been devoted by physicists and astronomers to the derivation of this formula as a physical law—to no avail. The futility leads to the question of whether it is not simply a statistical accident, as convincing as the results in the Table might be. The statistician Good (1969) concluded that there was only one chance in 130 that it was a statistical fluke. But another well-known statistician (Efron, 1971) inferred that the probability was about 50% that it was indeed a statistical accident. Without attempting to analyze the remarkable difference between these two conclusions (it appears to lie with differing null-hypotheses—see Huybers, 2004), it stands as an example both of the treachery of certain kinds of statistical inference, but also the ability of nature to provide intriguing patterns that are indistinguishable from chance.

A somewhat different form of pattern was the focus of attention by the well-known British physicist A. Eddington whose book (1946) described a number of seemingly important expressions for the fundamental constants of nature. For example, Lenz (1951) noted that the ratio of the proton to electron mass ratio was nearly $\mu_p/\mu_e = 6\pi^5$. Or (Wyler, 1969), the fine structure

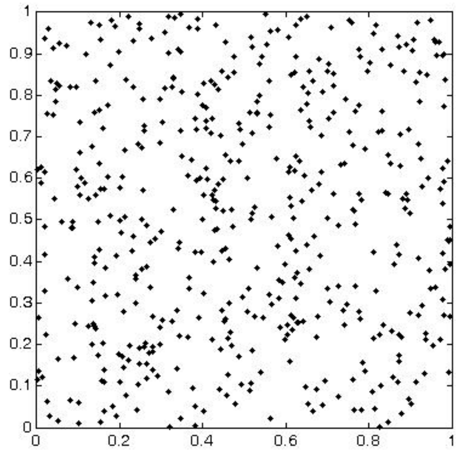

Figure 10: A "star-chart" generated by determining horizontal and vertical positions by drawing two numbers from uniform distributions in $[0,1]$. The eye seeks non-existent patterns. {Figurerandomd

constant by observation is $\alpha^{-1} = 137.03611 \pm 0.00021$. One then finds,

$$\alpha^{-1} = \left(9/8\pi^4\right)\left(\pi^5/2^4 5!\right) = 137.03608245, \tag{1}$$ {finestructure

which appears to call for explanation. But it turns out (e.g. Roskies, 1971) that there exists an infinite number of such formulae in small integers and π, e, and expressions such as Eq. (1) have no apparent physical significance, intriguing as they are.

5 Interesting Classroom Examples

In teaching it helps to suggest some of the pitfalls of superificial, non-objective, inference. Here are some examples intended to make a class think a bit.

- Consider Fig. 10 which resembles many star charts. Are these dots structured? From the earliest days of astronomy, people have been finding patterns in such pictures (viz., the constellations). The particular pattern shown is completely random, but one's eye is attacted to various clusters and one might even generate a theory of these patterns. For this reason, astronomers have long been concerned about the inference of spurious patterns (see Julesz, 1981; Barrow and Bhavsavar, 1987; Newman et al., 1994).

- A couple has two children. One of them is a girl. What is the probability that the other child is a boy?[2]

[2] Two-thirds. See Gauch (2003).

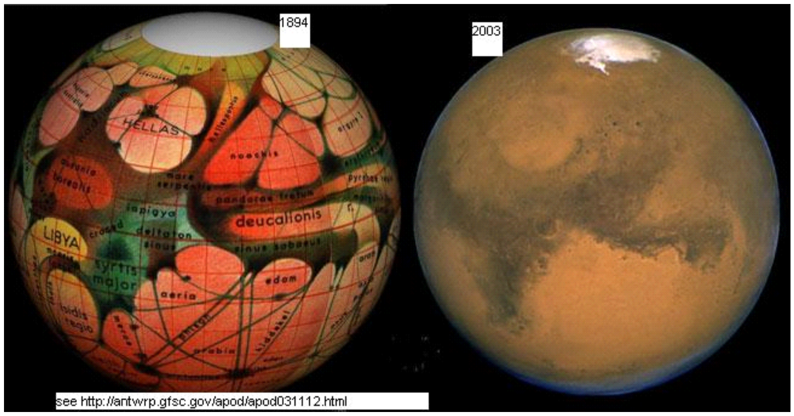

Figure 11: On the left is a view of Mars drawn in 1894 by Giovanni Virginio Schiaparelli and on the right is from the Hubbell Telescope (from NASA website).

{Figuremarscan

- Consider the game of Peter and Paul in which a true coin is flipped sequentially. Every time the coin comes up heads, Peter pays Paul $1, and Paul pays Peter the same amount if the coin is tails. Draw a picture example of Paul's winnings through time.[3]

- In a game, 20 marbles are distributed randomly among 5 players. Two example outcomes are:

Player: A B C D E

Game 1: 4 4 5 4 3 Type I outcome (non-uniform)

Game 2: 4 4 4 4 4 Type II outcome (uniform)

Which is more probable in the long run? [4] (Related to the so-called law of small numbers—the common expectation that small samples should display the statistical properties of a large-sample.).

- Basketball player A tends on average to make 30% of his shots. Player B averages 35%. Player A has just made 8 shots in a row. Player B has missed his last two. Should you bet on player A or player B?[5]

- Cause and effect inference confounded by tendency of regression toward the mean: Anecdote of pilot school—instructors find that when giving praise for a good landing, the next time out, the student seems often to do worse. But when, instead, a bad performance

[3] Feller (1957)

[4] The second is more probable. Kahneman et al., (1982) P. 36

[5] A run of eight shots in a row *must* occur by chance. Empirically, the idea of a "hot hand" in basketball cannot be distinguished from happenstance and on average one is advised to nonetheless bet on player B.

is harshly criticized, the next time out, the student does better. Pedagogical conclusion is that one should not praise, only criticize! A statistical test shows the difference is no greater than expected from chance.[6]

- A population of women is known to have a probability of 1 in 100 of having breast cancer. One woman has a mammogram which shows a lump, believed malignant. It is known that when a tumor is present, the test has an 80% accuracy (that is, shows as malignant). When no tumor is present, the test shows a false positive 10% of the time. What is the probability that the woman has cancer?[7]

- The Monte Hall Game—more formally known as Bertram's (1889) paradox. Named for the host of the television quiz show "Let's Make a Deal." Contestant is faced with three doors. She is told that behind one door is a car, and behind the other two doors are goats. Contestant is asked to pick a door—which is not opened. Host (who knows where the car is) then opens one of the two remaining doors to show a goat. Contestant is then asked if she wishes to switch her original choice to the remaining closed door. Should she switch?[8]

6 Discussion

The moral of the story is that statistical and probabilistic inference needs to be done carefully, with as many assumptions the investigator is aware of, being made plain and explicit (National Research Council, 2006). There are many ways to get into trouble, but in general, careful use of existing statistical methods, transparency, and lingering skepticism are safe harbors for the scientific investigator. Statisticians sometimes remind people of the words of Oliver Cromwell to the Church of Scotland (J. Kadane, private communication, 2007): "I beseech you, in the bowels of Christ, think it possible you may be wrong" (Bartlett, 1968). Locale aside, it seems like good advice for scientists.

[6] Kahneman et al. (1982). It was suggested at the meeting that one should criticize successful landings and applaud bad ones. Presumably that experiment has not been conducted.

[7] Eddy (1992). From Bayes's theorem, the correct answer is 8%.

[8] She should switch as there is a 50% chance that the car is behind the other door. See http://math.ucsd.edu/~crypto/Monty/montybg.html. Also Wikipedia which has an extended discussion.

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