

# De-Aliasing of Global High Frequency Barotropic Motions in Altimeter Observations

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**Abstract.** The existence of high frequency (periods shorter than 10 days) energetic barotropic motions in the ocean is shown to lead to a large aliasing error in satellite altimetric observations. This error is most easily seen in the “trackiness” of 10-day altimetric maps, and commonly attributed to orbit error. Fortunately, existing ocean general circulation models, when driven with twice-daily windstress fields, have considerable skill in predicting these motions. With improved forcing and models we can expect that in the future the alias can be largely suppressed by subtracting the model-generated high frequency fields.

## Introduction

The ocean supports fluctuations on all time and space scales, and the need to adequately sample them on a global basis provided much of the justification for recent altimetric satellite systems. These space missions were designed under the assumption that with the exception of a few special places (particularly near western boundary currents and near the equator), little difficulty would be experienced from the presence of energetic high frequency motions. Satellite altimeters have indeed proved extremely useful in thousands of published applications over the past 10+ years.

Contrary to the inferences of *TOPEX Science Working Group* [1981], *Fukumori et al.* [1998, hereafter FRF98] called attention to the presence in their model of a surprising amount of high frequency, barotropic motion, which appeared particularly intensified in two regions of the Southern Ocean, and in the northern North Pacific. They showed some apparent covariance of the model motions with measurements from the TOPEX/POSEIDON (T/P) spacecraft.

The FRF98 results imply the possibility that aliasing from very high frequency motions may be seriously corrupting the altimetric measurements. In this present note, we seek to explore explicitly the question of the extent of aliasing present in the T/P data, and whether it is correctable from predictions of existing numerical circulation models. Answers to these questions have strong implications for the planning of new altimeter missions. The questions of aliasing are made even more urgent because of the imminent

launch of time-dependent gravity measuring missions [*Wahr et al.*, 1998] which would experience yet greater difficulties. If significant high frequency motions are present in the ocean, both high-quality altimetry and precise gravity missions will require their removal. (*Tierney et al.*, 1999 have come independently to nearly the same conclusions.)

## Predicted Barotropic Motions

To extend the FRF98 results, we use the MIT ocean general circulation model (GCM) [*Marshall et al.*, 1997]. In a variety of numerical experiments, which will be reported elsewhere, we have found that significant high-frequency motions at periods shorter than about one month exist in the full baroclinic MIT primitive equation GCM. The model was run on a 1°x 1° geographical grid and was driven by twice-daily wind stress from the National Center for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) reanalysis project, and daily NCEP buoyancy fluxes.

In addition, a barotropic version of the same model was run with identical wind forcing, which has uniform density but otherwise satisfies the same primitive equations. In contrast to the baroclinic version, the barotropic model has only one layer, but treats the varying bottom topography with a lopped-cell formulation [*Adcroft et al.*, 1997]. The experiments showed that much of the rapid motion present in the full baroclinic model could actually be simulated to a high degree of accuracy with this simplified barotropic version of the model, which for operational applications can greatly reduce the computational burden.

Because results from the two models are so similar at periods shorter than about one month, we will focus on the high-frequency motion of the barotropic one. This model was run from 1985 to 1997 (the spin-up time is about 4 weeks) and fields were sampled and stored four times per day. A map of the resulting rms surface pressure variability (the same here as the rms bottom pressure variability) is shown in Fig. 1. Consistent with FRF98, we see conspicuous regions of intense variability northwest of the Drake Passage, southwest of Australia, and to a lesser extent in the high latitude North Pacific and Atlantic Oceans, but there are numerous regional differences.

## Sampling Issues

Fig. 2a shows a typical timeseries of the model pressure field, sampled 4 times per day, from a location at 257.5° E, 51.5° S, in which changes of up to 20 cm occur within a few

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days. With a repeat cycle such as the 10-days of T/P, all components with periods shorter than the Nyquist period of 20 days will alias into longer periods unless a complicated wavenumber filter can be constructed to suppress the energy in the spatial domain. Global frequency/wavenumber spectra show that on average about 50% of the total sea surface variability lies at periods shorter than 20 days, and is a measure of the possible aliasing error. The red-line in Fig. 2a shows that the full time series at this position would be grossly undersampled by the T/P spacecraft. Fig. 2b shows a comparison of the resulting model barotropic pressure fluctuations with those observed by T/P at the same positions. The T/P data have been processed conventionally. This processing includes the inverted barometer correction, discussed below. The agreement is surprisingly good (the correlation coefficient is 0.6).

## Demonstrating Skill

Figure 2 shows explicitly that over at least part of the ocean, a barotropic model driven by weather center-provided twice-daily wind stress fields has substantial energy at periods shorter than 20 days; predicts much of the high-frequency variability that is actually observed by T/P; and has sufficient skill to remove much of this high frequency motion from the altimeter data. The possibility of a strong aliasing effect does not seem to have been widely recognized previously and significant implications exist for future altimetric and gravity data handling strategies.

To demonstrate the model skill, we used the model output to actually predict the surface elevation seen by T/P, as in Fig. 2a, but over all repeat cycles 2 through 195 (5.3 years). Fig. 3a shows a global map of the point-wise in-time correlations of the model sea surface variability with that from T/P. Apart from a few, primarily tropical, regions of small negative values, where signals are very small and baroclinic effects are more important, the correlation coefficient is generally positive and often quite large. Fig. 3b displays a spatial map of the percentage variance reduction of the T/P data after applying the model-generated correction over repeat cycles 2-195. At high latitudes, and particularly at the most energetic sites, the variance reduction is considerable — up to 30% of that in the original T/P data.

## Correcting TOPEX-POSEIDON Data

In the following, we will take the novel step of correcting the T/P data for barotropic motions by subtracting the model predictions from the along-track elevation. Figure 4 shows the field of the simulated motion after sub-sampling along T/P tracks during one 10-day orbit cycle and their subsequent gridding on a  $2^\circ$  by  $2^\circ$  spatial grid. There is a very strong “trackiness” in the gridded fields — suggesting large motions which are uncorrelated between neighboring satellite passes. This trackiness is well-known in altimetric data, and until now has been attributed to errors in the spacecraft orbit estimates. We suggest here that, at least for the T/P mission with its much improved orbits, much of the observed trackiness is instead present owing to rapid barotropic fluctuations in the sea surface topography and reflects a sampling problem of the true ocean variability, rather than just orbit error.

To support this statement, we display in Fig. 5a an estimate of the power density spectrum of alongtrack barotropic

$\eta'$  data as predicted by the model for 20 individual repeat cycles. In Fig. 5b we show similar spectra, but from the original T/P height anomalies (red) and from the corrected time series (blue) from which the barotropic model component was removed. The model and original T/P surface elevation anomaly spectra show a considerable degree of agreement in their overall shapes. In particular, both show enhanced amplitudes at a frequency near once-per-orbit period (6732s) and at its higher harmonics. At most of those frequencies, the spectrum of the barotropic motion-corrected height anomaly displays diminished power. The total variance reduction in Fig. 5b is from  $81 \text{ cm}^2$  to  $77 \text{ cm}^2$ . Subtracting the model prediction from the T/P surface elevation anomalies indeed removes much of the original trackiness. This correction, although still incomplete, is almost as large as the inverted barometer correction and dominates the remaining error terms.

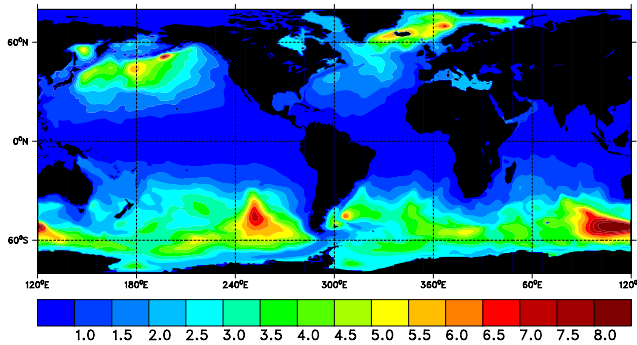
## Towards an Operational Correction

Our conclusion is that high frequency barotropic contributions to both altimetric and future temporal gravity measuring missions are a significant signal, and that their first-order effect is to alias into the measurements. Fortunately, existing GCMs, either barotropic or baroclinic, when driven by center-provided wind stress fields, show skill in calculating and thereby partially suppressing the aliasing, by subtraction from the measurements. We propose therefore that the undertaking of such an operational use is an urgent next step for ongoing and future altimetric and gravity projects.

One may not wish to subtract energy at periods longer than the mission Nyquist period that is both well-represented in the model, and adequately sampled by the spacecraft. It is an easy matter to high-pass filter the model output to remove such components from the correction.

There are several issues which we do not have space to address, but which need serious consideration as improvements. No attempt has been made to optimize the model for the present purpose and we anticipate that modified bottom topography, frictional coefficients, etc. should significantly improve the model skill. As with all models, the skill necessarily varies with region. The extent to which a high resolution baroclinic model generating a full eddy field would produce a qualitatively changed high frequency signal is unknown and must be explored.

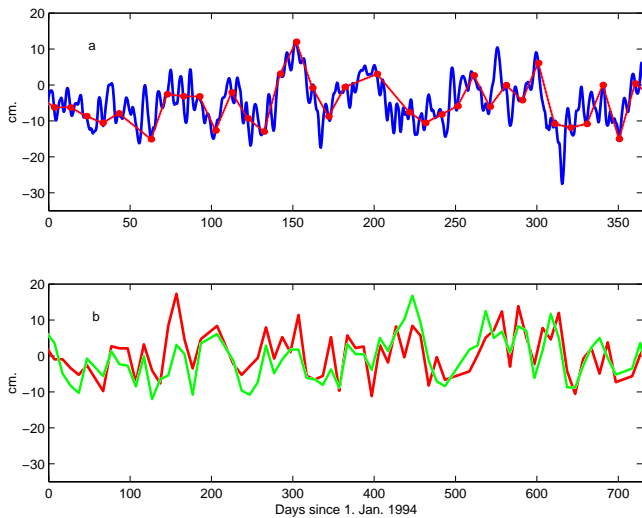
Only wind-driven sea level signals have been considered here, and the static inverted barometer response was assumed to be valid. We have not tried to account for any dynamic sea level signals forced by atmospheric pressure. Experiments with other models [*Ponte and Gaspar, 1999*] show that at periods shorter than a few days a significant dynamical pressure-driven response can occur (see the review by *Wunsch and Stammer, 1997*). This response can be larger than wind-driven signals at sub-weekly timescales [*Ponte, 1994*], and will contribute to the aliasing problem. Although the impact of including full pressure-driven effects in reducing variance in altimeter records needs to be carefully evaluated and requires further model development activities, it seems that, in operational use, models must include both wind- and pressure-forced contributions. But the quality of high frequency atmospheric forcing fields is a concern.



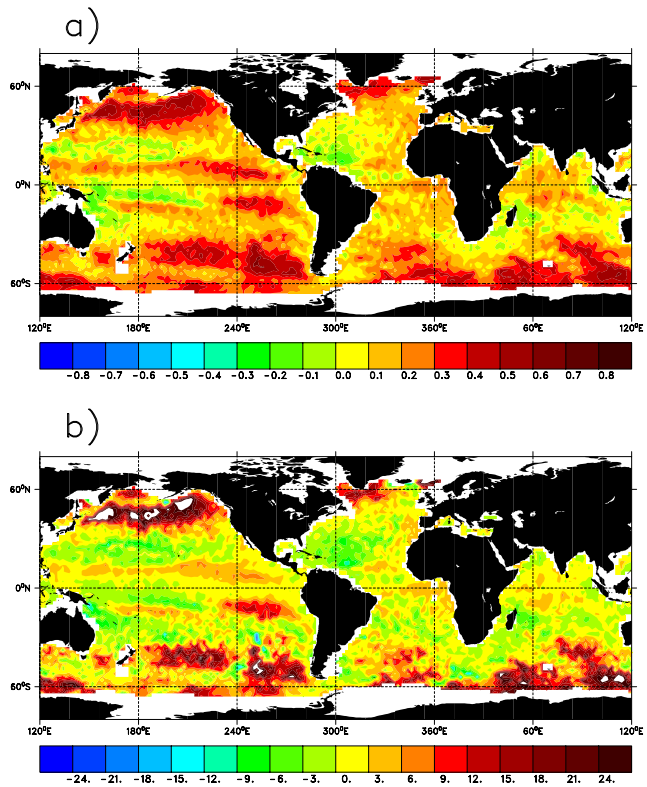
**Figure 1.** (a) Root-mean-square (rms) seasurface height variability (in centimeter) from the barotropic model sampled four times per day over the interval 1993 to 1997.

**Summary**

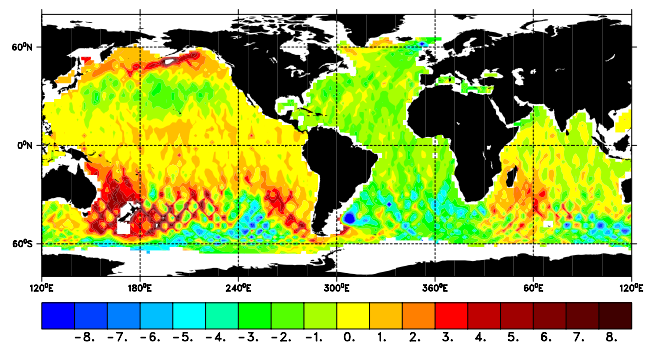
A barotropic GCM predicts significant aliasing of energy into the pressure fields (surface or bottom) of the ocean if the sampling interval is as infrequent as once every few days. The model is shown, by comparison to the T/P data which come from 10-day samples, to have considerable skill, but it can undoubtedly be improved. The major conclusion is both that the aliased energy is important, particularly at high latitudes, and that it can be partially removed using the existing generation of numerical models. After the tides and the inverted barometer, high frequency barotropic motions are the most significant error in the altimetric record. As with the tides and tide models during the T/P mission, we expect not only that these models will improve greatly, but that the physics of the barotropic motions will be the focus of considerable attention.



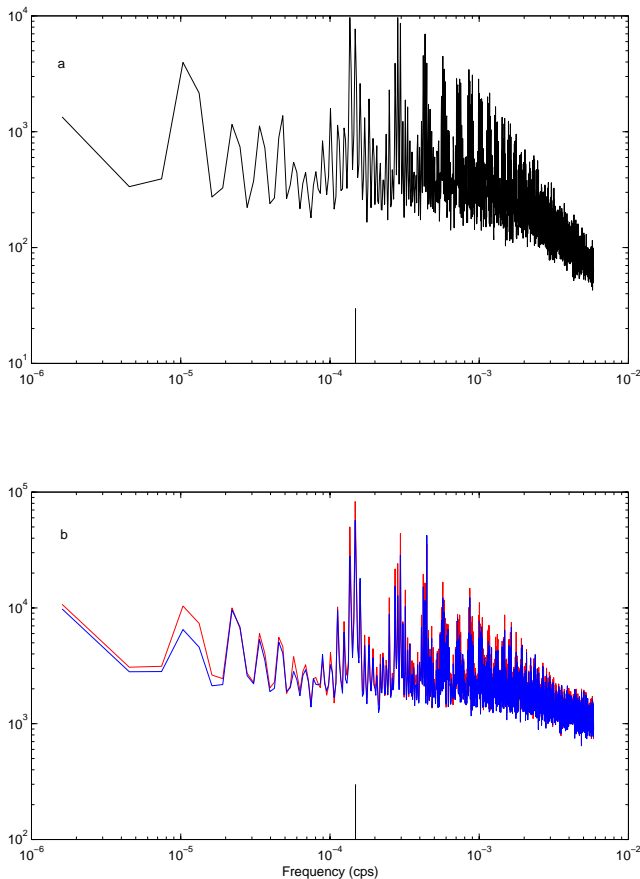
**Figure 2.** (a) Model elevation anomaly  $\eta'$  (in centimeter) sampled 4 times per day at 257.5°E, 51.5°S (blue curve), and as sampled only once every 10 days during TOPEX/POSEIDON overflights (red dots). The figure displays the one-year period of 1994 from which the strong aliasing is obvious. (b) Model surface elevation sampled at TOPEX/POSEIDON times (red curve) and actual TOPEX/POSEIDON observations (green curve), both shown here for the 2-year period 1994-1995.



**Figure 3.** Correlation coefficient in time between model surface elevation and TOPEX/POSEIDON data. computed at each along-track position and gridded subsequently on a 2°x2° grid. (b) Percentage variance reduction in TOPEX/POSEIDON surface elevation (in centimeter<sup>2</sup>) over the time span of repeat cycles 2 through 195, after the model correction was removed from the data.



**Figure 4.** Model elevation, sampled along TOPEX/POSEIDON tracks and gridded subsequently on 2°x2° grid for repeat cycle 30.



**Figure 5.** (a) Spectrum for the along-track model surface height anomaly. In (b) a similar spectrum is shown for the along-track TOPEX/POSEIDON surface height anomaly,  $\eta'$  (red curve) and the corrected TOPEX/POSEIDON surface height anomaly after subtraction of the model-predicted values of  $\eta'$ . Notice the large reduction in energy near one cycle per orbit. The spectral algorithm was that of Lomb and Scargle, which is discussed by Press *et al.*, 1992. The once-per-orbit frequency is marked in both panels.

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