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Global Ocean Integrals and Means, with Trend Implications

Carl Wunsch

Department of Earth and Planetary Sciences, Harvard University, Cambridge, Massachusetts 02138; email: cwunsch@fas.harvard.edu

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Abstract

Understanding the ocean requires determining and explaining global integrals and equivalent average values of temperature (heat), salinity (freshwater and salt content), sea level, energy, and other properties. Attempts to determine means, integrals, and climatologies have been hindered by thinly and poorly distributed historical observations in a system in which both signals and background noise are spatially very inhomogeneous, leading to potentially large temporal bias errors that must be corrected at the 1% level or better. With the exception of the upper ocean in the current altimetric-Argo era, no clear documentation exists on the best methods for estimating means and their changes for quantities such as heat and freshwater at the levels required for anthropogenic signals. Underestimates of trends are as likely as overestimates; for example, recent inferences that multidecadal oceanic heat uptake has been greatly underestimated are plausible. For new or augmented observing systems, calculating the accuracies and precisions of global, multidecadal sampling densities for the full water column is necessary to avoid the irrecoverable loss of scientifically essential information.

1. INTRODUCTION

The ocean contains Earth's dominant reservoirs of heat, freshwater, carbon, energy, etc., in near equilibrium with the atmosphere, cryosphere, lithosphere, and biosphere. The magnitudes of the reservoir changes in these properties that arouse both intense societal anxiety and scientific interest are minute fractions of the background exchanges and reservoir totals. How does one measure and understand a global fluid and its slight deviations from near equilibrium? And how accurately must this be done?

Historically, physical oceanographers took an essentially pragmatic approach of measuring what they could, where and how they could, with little attention paid to long-term goals and quantitative requirements. Depictions came from ad hoc combinations of data that were either sketchily global (with the RRS *Challenger* survey as a prototype) or synthesized from various regional efforts (e.g., Montgomery 1958, Reid 1981, and the various textbooks of oceanography that appeared beginning in the early twentieth century). It would have been an unusual hydrographer who in 1960 would have invested in difficult absolute calibrations so that successors in 2020 could calculate mean oceanic salinities significant at one part in 10^6. Goals were often local (perhaps determining regional geostrophic shear, or depicting an eddy) and short term. Similarly, a decision in 1980 to move a tide gauge from one pier to another within a harbor would have likely involved only a discussion of local tide prediction accuracy, and not its utility for calculating global mean sea level changes.

The importance of this subject lies in its consequences for future generations. Anyone who has grappled with reconstructing the state of the ocean in past decades is frustrated by the inadequacy of earlier observations. The present generation of oceanographers must ensure that its successors will not be equally frustrated, decades in the future, by a failure to understand what is required and what could have been readily done.[1]

In the past three or four decades, physical oceanography has progressed from a small academic subject to one in which journalists, bloggers, and tendentious politicians commonly pounce on technical findings about sea level change, ocean pH, heat content, "hiatuses," and the like. Combined with funding needs and a common liking for celebrity, the result has been a sometimes dispiriting literature and debate. Dubious claims to accuracy abound, ones that often imply the adequacy of a remarkably small number of observations.[2]

Systems for which budgets for energy, mass, etc., remain unbalanced are not fully understood. Prediction (what society seeks) then rests on a flimsy foundation. A great deal is known about

[1]The intergenerational nature of climate science has been discussed by Baker et al. (2007) and Wunsch et al. (2013). Questions of how much to invest today in measurement systems that may produce scientifically exciting data only after many decades is analogous to debates by economists concerning discounting of the future (e.g., Weitzman 1998). A few scientists have been well aware of the problem. Consider Nansen's (1902, p. 3) comments: "It is, in my opinion, a matter of much regret that the oceanic research of our day has not kept pace in accuracy with the development of physical determinations. This is especially true with regard to deep sea temperatures, which are very far from satisfying our modern demands for accuracy. We have decidedly not done our best to utilize for oceanography the present high development of thermometry, which could enable us to measure temperature at least to within ± 0.01°C.; but this is still, as a rule, far from being the case. I fear that 50 years hence, our temperature measurements of the ocean of to-day will not be of much value, as they are not sufficiently accurate. This is so much the more to be regretted, as not only secular changes in the circulation of the sea, but possible secular variations in the temperature of the atmosphere or the surface of the earth can probably be most easily demonstrated by the variations of the mean-temperature of the ocean." See also Nansen's preface. (I am grateful to Greg Johnson for calling my attention to this quotation.)

[2]An additional problem is that anyone who has invested substantial time and energy in analyzing a complicated data set and wants to publish is driven to find a signal whether one exists or not, sometimes in the teeth of the authors' own uncertainty estimates. For example, Allan et al. (2014) declared that global heating of 0.34 ± 0.67 W/m^2 from 1985 to 1999 and 0.62 ± 0.43 W/m^2 from 2000 to 2012 shows global warming. Global cooling would evidently also be consistent within their 90% confidence intervals.

how to obtain measurements almost anywhere in the ocean, and sustaining such observations is a solvable political problem. What has tended to be missing from much of the discussion is the quantitative aspects: How accurate and precise must the resulting measurements be? The questions arising from quantification of global measurement systems are taken up at the end of this review.

Many reasons exist for seeking the characteristics of a long list of oceanic physical variables, region by region, basin by basin, up to the global scale. To limit the scope of this review, I focus on aspects of the problem of quantifying global integrals and global averages, recognizing that doing so involves integration over all of the diverse subregions. The field is in a difficult transition from the semiqualitative (construction of horizontal salinity maps) to the fully quantitative (what is the mean oceanic salinity today, and is it known to 0.001 g/kg?).

Complicating this transition has been the rise of the ocean/climate modeling culture, in which computing over long time periods and comparing models are easy compared with making measurements. I do not attempt to evaluate the large and contradictory literature on unconstrained model calculations of possible oceanic change.

2. WHY BOTHER?

Let an overbar denote a time average, and a bracket, $\langle \cdot \rangle$, a global average. For a number of central oceanic problems, the absolute value of a variable $\langle \overline{C} \rangle$ is of central interest: Why is the mean oceanic temperature 3.5°C, the mean salinity approximately 35.5 g/kg, and the range of mean surface height, $\bar{\eta}$, 3 m and not 30 m? Why is the kinetic energy of the general circulation approximately 14 EJ and not ten times or one-tenth that? How different could they be? As the science now stands, great accuracy is not required either for these numbers or for defining their temporal averaging interval, as theory in some cases is still coping with explaining factors of two or larger. Much more demanding is the study of change in the time-dependent global averages, $\langle C(t) \rangle$, and the possibility of real and interpretable trends in those changes.

Studies of temporal change must be based on the observed determination of $\langle C(t_\mathrm{p}) \rangle$ at one or more times, t_p, or, depending on various assumptions, over some time interval, $\Delta t(t_\mathrm{p})$, about t_p. The problem thus divides partially into issues of determining first the space-time means and then those related to temporal changes.

2.1. Fundamental Science and the Global Budgets

Much of science rests on a foundation of conservation laws, including those for mass, energy, angular momentum and vorticity, and the like. The ability to close the global budgets for heat (enthalpy), kinetic energy, freshwater, organic carbon, oxygen, etc., becomes a test of whether Earth and its subsystems are fundamentally understood—or otherwise.

2.2. Understanding Regional Phenomena and Their Variations

Many, and perhaps most, oceanographic and related societal problems are dominantly regional in nature. Few people experience global mean sea level rise or global salinity changes. Many of these regional shifts can, on some timescale, be understood as arising from similarly localized, recent causes. A change in the local wind curl can drive an observable local sea level change, and an increase in the transport of a relatively fresh current influences the local salinity and hence the sea level and biology. On the other hand, a central characteristic of a fluid is that all regions are ultimately in contact with, and influenced by, all other regions. Time delays might be long, and/or the magnitude of the influence might be imperceptible, but neither can be neglected by fiat. Indeed, a demonstration that some large part of the ocean never influenced any other region would be an extremely useful theoretical result.[3] The Navier-Stokes and thermodynamic equations imply nonzero space-time covariances everywhere and for all times, including ones with arbitrarily long delays. An inference of a global change can be amplitude dependent: Physics demands that a persistent local sea level change of 10 m be a global phenomenon, but a shift of 10 cm can be either local or global.

2.3. Modeling Needs

Models are the essential synthesis tools for observational data. In practice, they require input fields and parameterizations on a global basis. These range from the almost mundane but not always adequately known (the bottom topography) to complex, sometimes plausible parameterizations of unresolved scales. Interpreting in a real-world context calculations from fluid models, uncalibrated on the spatial and temporal scales they portray, takes place on a slippery slope.

Making observed results useful in almost any context involves several aspects:

1. For any measurement of known type, what are the errors, including biases and stochastic elements?
2. With measurements at known times and positions, with known errors, what is the best way to calculate the global integrals or areal or volume averages? How reliable are the results?
3. How are integrals and averages of known accuracy and precision best interpreted in terms of ocean physics?

In discussing future observations, new questions arise:

4. What is the best mix of instruments and space-time sampling strategy to meet any particular goal?
5. How does one sustain such efforts, including assuring ongoing, long-term calibrations?

Questions 4 and 5 can be addressed only in the context of questions 1–3—the focus here. Specific examples are drawn from the fields that have been most discussed: temperature (heat), salinity (freshwater and salt content), sea level, and energy, variables that are inseparably entwined through the influence of both temperature and salinity on the elevation, η, with altimetric data also dependent on ocean dynamics. The intention is not to add to the ballooning literature on how oceanic heat content has changed (see Antonov et al. 2005, Ishii & Kimoto 2009, Purkey & Johnson 2010, Lyman & Johnson 2014, and Roemmich et al. 2015, among numerous others, varying by time interval, water depth, and methodology), trends in sea level (Church & White 2011, Stammer et al. 2013), etc., but only to describe the context in which the competing estimates and proposed future observations must be evaluated.

3. REQUIRED ACCURACIES AND PRECISIONS

As already noted, for basic scientific reasons, sometimes even crude measures of time-average properties are of intense interest. Going beyond such measures to determine changes in oceanic properties is difficult because of the slight expected values relative to the background state. As an example, **Table 1** lists the correspondences between oceanic heating rates over a decade and the corresponding changes in global mean temperatures. For context, estimates of the increment in heating of the oceans by greenhouse gas increases are of order 0.5 W/m^2 (e.g., Stephens et al. 2012),

[3]The late J.L. Reid of Scripps Institution of Oceanography was fond of describing his work as "the study of the California Current and all of its tributaries," an elegant way of saying the ocean is global.

Table 1 Approximate oceanic temperature changes implied by a 1 W/m^2 heating (or cooling) rate over different times and depths, as well as the temperature change equivalent of a 1 mm/year global mean sea level (GMSL) change

Period and fraction of water column	Temperature change for 1 W/m^2 heating/cooling rate	Temperature change for 1 mm/year GMSL change
1 year, full depth	0.002°C	0.0015°C
10 years, full depth	0.02°C	0.015°C
1 year, upper 700 m	0.01°C	0.008°C
10 years, upper 700 m	0.1°C	0.08°C
1 year, below 700 m	0.0025°C	0.002°C
10 years, below 700 m	0.025°C	0.02°C

For scaling purposes, the heat capacity $c_p = 4{,}000$ J/(kg·°C), $h = 3{,}800$ m, and $\rho = 1{,}038$ kg/m^3. The expansion coefficients α are in the range 5–30 $\times$ 10^{-5}/°C (Thorpe 2005) and are smaller—even negative, for freshwater—near the freezing point. A GMSL change of 1 mm/year corresponds to a volume flux of approximately 1.1 $\times$ 10^4 m^3/s = 0.01 sverdrups. Modified from Wunsch & Heimbach (2014).

which would correspond to a mean temperature change of 0.01°C in a decade. An old rule of thumb is that a measurement's accuracy should be no worse than approximately 10% of the expected signal. In the present case, and noting that longer durations produce larger expected temperature changes, a reasonable inference is that measuring anthropogenic global mean change requires an accuracy of 0.05 W/m^2 or a mean temperature equivalent change of 0.001°C/decade. (Regional signals are much larger, with much relaxed accuracy requirements for their local detection.)

Estimates of global mean sea level change are generally $\langle \Delta h \rangle / \Delta t \approx 3$ mm/year (e.g., Stammer et al. 2013). By the same 10% rule, measurement accuracies equivalent to 3×10^{-4} m/year are needed. That corresponds to a global mean temperature change of (very roughly) 0.0015°C/decade, if the sea level rise is attributed solely to temperature change (**Table 1**). (The thermal expansion coefficient of seawater is strongly temperature dependent and hence also regionally dependent.) If, instead, the sea level change is wholly due to freshwater addition or subtraction, the corresponding volume flux would be approximately 0.003 sverdrups (Sv), corresponding to a change in oceanic salinity, S, of

$$\frac{\langle \Delta S \rangle}{S_0} = -\frac{\langle \Delta h \rangle}{h_0} \tag{1}$$

(see Wadhams & Munk 2004), where $S_0 \approx 35.5$ g/kg and $h_0 \approx 3{,}800$ m, or $\langle \Delta S \rangle \approx 3 \times 10^{-6}$ g/kg per decade. These rough numbers are reference points.

To the extent that only temporal changes are of interest, the focus shifts to precision rather than accuracy. Consider two estimated means, $\tilde{m}(t_{\mathrm{I}})$ and $\tilde{m}(t_{\mathrm{F}})$, over two time periods, δt_{I} and δt_{F}. The estimated difference $\Delta \tilde{m} = \tilde{m}(t_{\mathrm{F}}) - \tilde{m}(t_{\mathrm{I}})$ then subtracts any common systematic errors. On the other hand, in a field like oceanography, subject to major technological and space-time sampling changes over decades, systematic errors are unlikely to be subtractive. If the errors in averages at the two times are $\varepsilon(t_{\mathrm{I}})$ and $\varepsilon(t_{\mathrm{F}})$, respectively, and are best regarded as statistically independent, then the accuracy of the difference of two such averages is $\varepsilon_{\mathrm{d}} = \sqrt{\varepsilon(t_{\mathrm{I}})^2 + \varepsilon(t_{\mathrm{F}})^2} > [\varepsilon(t_{\mathrm{I}}), \varepsilon(t_{\mathrm{F}})]$.

3.1. Sampling, Averaging, and Integrating Problems

Obtaining useful large-scale oceanic averages and integrals involves three primary elements: the data, the governing physics, and statistics of both the signal and noise fields. Historically, the global ocean has been extremely undersampled. Even today, with a revolutionary improvement in the last decade in direct upper-ocean observation systems, data from the abyssal regions (below 1–2 km) remain sparse. Attempts to nonetheless obtain useful averages have thus commonly made strong assumptions about the appropriate space-time statistics in an attempt to overcome the paucity of observations, and have in parallel deemphasized the governing physics. But no statistical magic can substitute for missing data—data are required to test and calibrate the statistical assumptions.

Measurements, y, always contain noise, usefully divided into a bias error, $\mathcal{E}(y - y_c)$, and a random element, $\mathcal{E}(y - \langle y \rangle)$, where $\mathcal{E}$ is the theoretical expected value[4] and y_c is the correct value being sought. Let the measurement at any time or place be denoted as

$$y_i = m + \varepsilon_i, \tag{2}$$

where m is the true mean value (over both time and space) and ε_i is the error, comprising both a bias, $\beta_i = \mathcal{E}(\varepsilon_i)$, and a stochastic component, $\sigma_{\varepsilon i}^2 = \mathcal{E}\{[\varepsilon_i - \mathcal{E}(\varepsilon_i)]^2\}$. For most oceanographic problems, both β_i and $\sigma_{\varepsilon i}^2$ are space and time dependent and the index, i, must be retained in them (in statistical jargon, y is heteroscedastic).

The best estimated value, $\tilde{m}$, is sought so that the spatial integral is $\tilde{m}_A A$ and the volume integral is $\tilde{m}_V V$, where A and V are the global oceanic areas and volumes, respectively, and the definition of $\tilde{m}_{A,V}$ varies as appropriate, with subscripts usually being omitted here. Summation operators, L, on y_i are sought such that

$$\tilde{m} = L(y_i),$$

so that the bias in $\tilde{m}$, $\beta_{\tilde{m}}$, is $\mathcal{E}(\tilde{m} - m) = 0$, and such that the uncertainty, measured as the variance, $\sigma_{\tilde{m}}^2 = \mathcal{E}[(\tilde{m} - m)^2]$, is a minimum.[5] Note that y_i will likely be regionally weighted by area or volume, as $y_i' = a_i y_i$, to reflect relative contributions.

Classical estimation theory (e.g., Liebelt 1967, Bretherton et al. 1976, Seber & Lee 1977, Wunsch 2006) acknowledges the potentially strong spatial variations in the variances, σ_ε^2, and the expected correlations of ε_i in both space and time. Employing no prior information about the magnitude of m, minimum variance estimation and its first cousin, ordinary least squares, produce an L such that

$$\tilde{m} = ([1, 1, \ldots, 1]\mathbf{R}^{-1}[1, 1, \ldots, 1]^T)^{-1}\mathbf{R}^{-1}\mathbf{y}', \tag{3}$$

with variance

$$\mathbf{P} = \frac{1}{[1, 1, \ldots, 1]\mathbf{R}^{-1}[1, 1, \ldots, 1]^T}, \tag{4}$$

where the noise covariance is $R_{ij} = \mathcal{E}\{[\varepsilon_i - \mathcal{E}(\varepsilon_i)][\varepsilon_j - \mathcal{E}(\varepsilon_i)]\}$ and $\mathbf{y}'$ is the vector of all weighted observations. (Corresponding expressions in the references also include Bayesian-equivalent solution statistical priors.)

In the limit that $R_{ij} = 0$, $i \neq j$ (uncorrelated noise),

$$\tilde{m} = \frac{1}{1/\sigma_1^2 + \cdots + 1/\sigma_M^2} \sum_{i=1}^{M} \frac{y_i'}{\sigma_i^2} \tag{5}$$

[4]Theoretical expectation is defined via the integral over the underlying probability density. Estimation via sample averaging relies on theorems for stationary and ergodic processes.

[5]Although unbiased estimates are desirable, in some cases acceptance of a finite bias is usefully traded for a great reduction in σ_m^2 (see, e.g., Liebelt 1967, Bretherton et al. 1976, von Storch & Zwiers 2001, Wunsch 2006, and numerous others).

and the minimum variance is

$$\mathcal{E}[(\tilde{m}-m)^2] = \frac{1}{\sum_i 1/\sigma_i^2}, \tag{6}$$

with the least noisy measurements having the most weight. (For an application of this argument where the average would be obtained from one almost noise-free measurement, see Hughes et al. 2012.)

If the very strong a priori assumption is made of uncorrelated noise of constant variance, then Equation 5 reduces to

$$\tilde{m} = \frac{1}{M}\sum_{j=1}^{M} y'_j. \tag{7}$$

The variance would be

$$\sigma_{\tilde{m}}^2 = \mathcal{E}[(\tilde{m}-m)^2] \approx \frac{1}{M-1}\sum (y'_j - \tilde{m})^2, \tag{8}$$

recalling that Equation 8 is an asymptotic result, accurate only as $M \to \infty$. These results rely on the tendency through the central limit theorem for sample averages to be Gaussian.

3.1.1. Instrumental noise in $\boldsymbol{\varepsilon}$. Oceanic instrument accuracy and calibration have been the subject of a still-growing literature that is too large to review here. Warren (2008) and reviews by Abraham et al. (2013) and Atkinson et al. (2014) discuss temperature; Millero et al. (2008), Warren (2008), and Wang et al. (2013) provide starting places for salinity; and Ablain et al. (2009) and Fu & Haines (2013) describe satellite altimetry errors.

3.1.2. Instrument bias errors.[6] If the bias, $\mathcal{E}(\varepsilon_j) \neq 0$, is known, then this part of the error in $\tilde{m}$ and its significance can be studied relative to the estimated value of $\tilde{m}$. In calculations such as Equation 7, $y_j - \mathcal{E}(\varepsilon_j)$ replaces y_j. A sufficient mix of instruments or regional variations producing bias errors of mixed sign may permit them to be treated as effectively random in a global average.

3.1.3. Field observers. In evaluating the bias and random errors in data, a distinction must be made between the best situation (carefully controlled laboratory conditions) and the realities of the field, including the variations in experience and attention from human observers. An example of the latter concern was Worthington's (1981) decision to discard all historical salinity data except for measurements obtained by a small trusted group (Warren 2008). Whether other calculations that used those data are nonetheless trustworthy is one of the troublesome judgments required for this problem.

3.1.4. Physical noise in $\boldsymbol{\varepsilon}$. Included in the covariances R_{ij} are real random processes in the ocean. Their physics includes internal waves, geostrophic eddies, planetary and coastal waves of many types, and more. Comparatively sophisticated expressions for these noise elements exist, including the Garrett & Munk (1979) spectrum, estimates of internal tide values (Garrett & Kunze 2007), and the eddy field (Wortham & Wunsch 2013). **Figures 1–3** show the variance, the variance distribution in the wavenumber spectrum of surface elevation from altimetry (Xu & Fu 2012), and the variance distribution by frequency (Hughes & Williams 2010). Regionally, each of these

[6] Position error has historically been a major problem. (For a brief summary of navigation errors, see Warren 2008.) It is probably true, however, that this error is of secondary importance in global-scale integrals, and with the arrival of GPS capabilities, it may be negligible in most measurements obtained after around 1990.

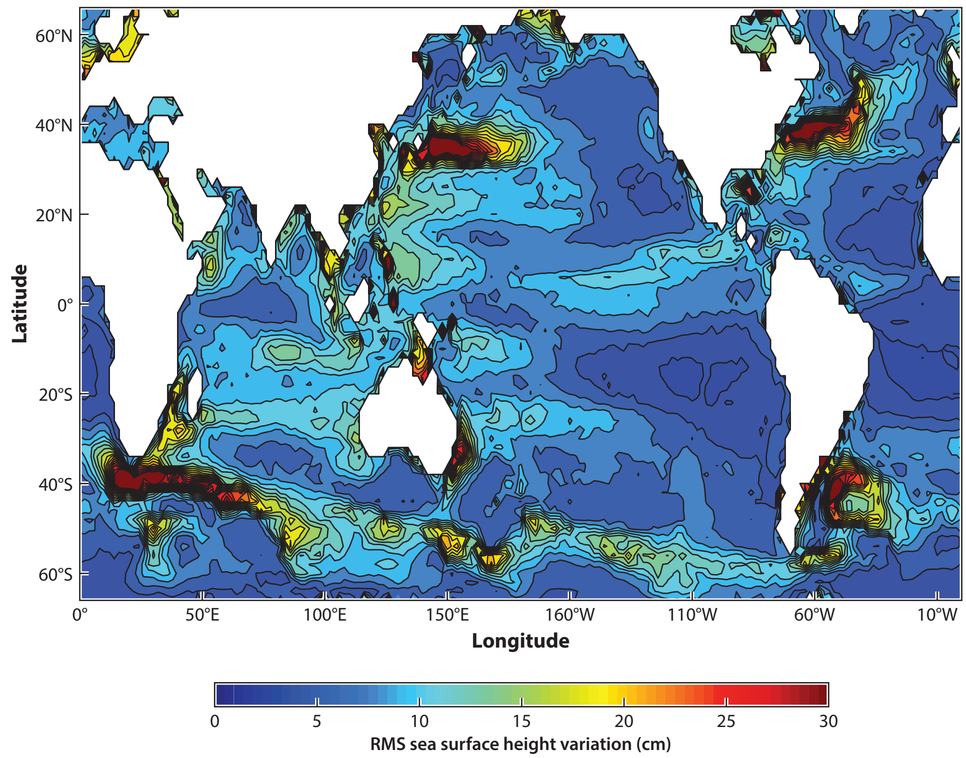


Figure 1

Root-mean-square (RMS) sea surface height elevation from altimetric measurements of η. Updated from Wunsch & Stammer (1998).

wavenumber and frequency power laws corresponds to a different spatial covariance matrix, **R**, but normal usage requires an ocean-wide continuous function of position and space-time lag (or wavenumber and frequency). Specifying **R** adequately is a complicated and still unsolved problem. (Wavenumber spectral estimates remain sensitive to assumptions about altimetric noise, particularly at the high end; see Zhou et al. 2015.) Altimetric noise is a global measure of temperature, salinity, and dynamical changes.

3.2. Inhomogeneities: Signal and Sampling

In addition to the space-time inhomogeneity of ε_i, two interlocking but distinguishable problems are encountered in oceanic computations of averages, $m_{A,V}$, or integrals, $m_A A$, $m_V V$:

1. Problem A: the very strong spatial inhomogeneity of oceanic physical regimes
2. Problem B: the very strong spatial and temporal inhomogeneity of historical oceanic measurement systems

The problem of how to best sample inhomogeneous media has a literature of its own, much of it focused on geological and mining problems (e.g., Myers 1997, Gy 1998). Although relevant here, complications intrude from the time dependence and the physics known from the equations of motion. Determining a best set of observation positions that are under one's control differs from deciding how best to use historical data. The oceanographic problem represents

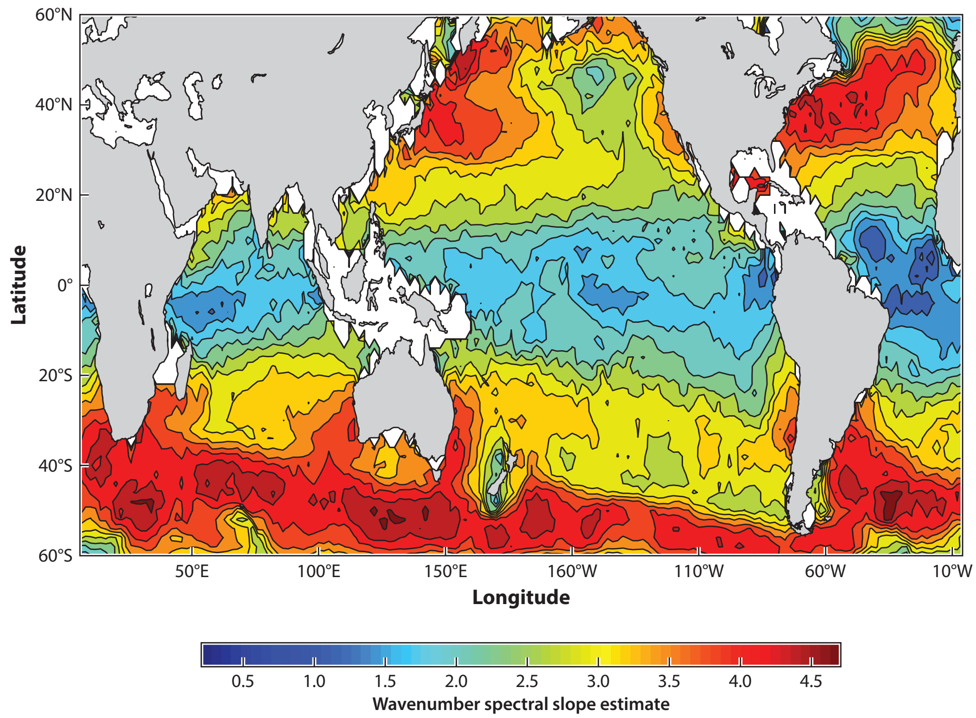


Figure 2

Power-law variations in altimetric variability kinetic energy spectra. Changes in values are a rough indication of the existence of differing dynamical regimes and of noise elements that must be accounted for in determining spatial mean values. Undersampled regions with power laws less steep than approximately −2 would be subject to significant spatial aliasing into the mean and other wavenumbers. Modified from Xu & Fu (2012).

the confluence of spatially highly inhomogeneous signals, background noise, measurement geography, and technological change.

As a specific example of the sampling problems, consider **Figure 4*a***, which is an estimate of the heat content, $\overline{H}(\lambda, \phi)$, of the oceanic water column as a 20-year average (Wunsch & Heimbach 2014); λ and ϕ are the longitude and latitude, respectively. For present purposes, this field is taken as the truth, as it is known to computer-word accuracy on a complete global grid, permitting the near-perfect calculation of mean temperatures, $\overline{T}(\lambda, \phi)$, in the form $\overline{H}(\lambda, \phi) = c_p V \langle \overline{T}(\lambda, \phi) \rangle$, where c_p is the mean heat capacity. Nearly all relevant data, including in situ temperature and salinity (with all Argo and elephant seal profiles), altimetry, and meteorological fields, with error estimates, were used in a weighted least squares fit of a model to the data. Good physical reasons exist to believe that this pattern has changed only in detail over the past 100+ years. Roemmich et al. (2012) addressed the weakly changed horizontal temperature gradients, Jevrejeva et al. (2008) showed that changes between long tide gauge records display decadal differences of at most tens of centimeters on a time-mean background of 3 m, and Kemp et al. (2011) bounded the changes on a millennial scale. Apart from external exchanges, any shift in the circulation will necessarily produce spatial changes in $\overline{H}$ without modifying the total or mean value (apart from equation-of-state nonlinearities). For measurement context, **Figure 4*b*** shows the 20-year temporal standard deviation of $\overline{H}$.

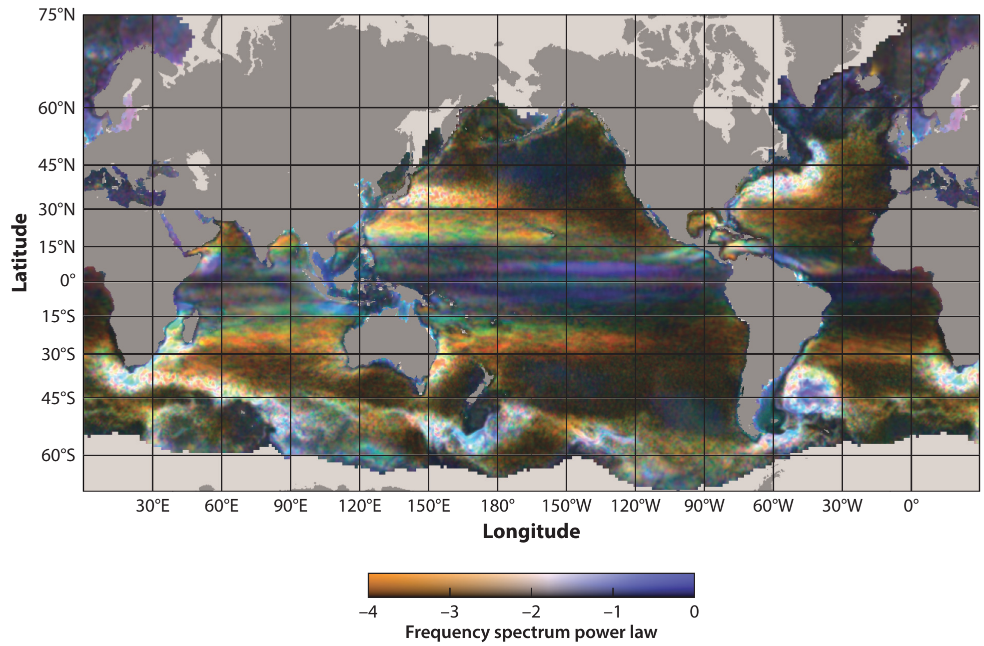


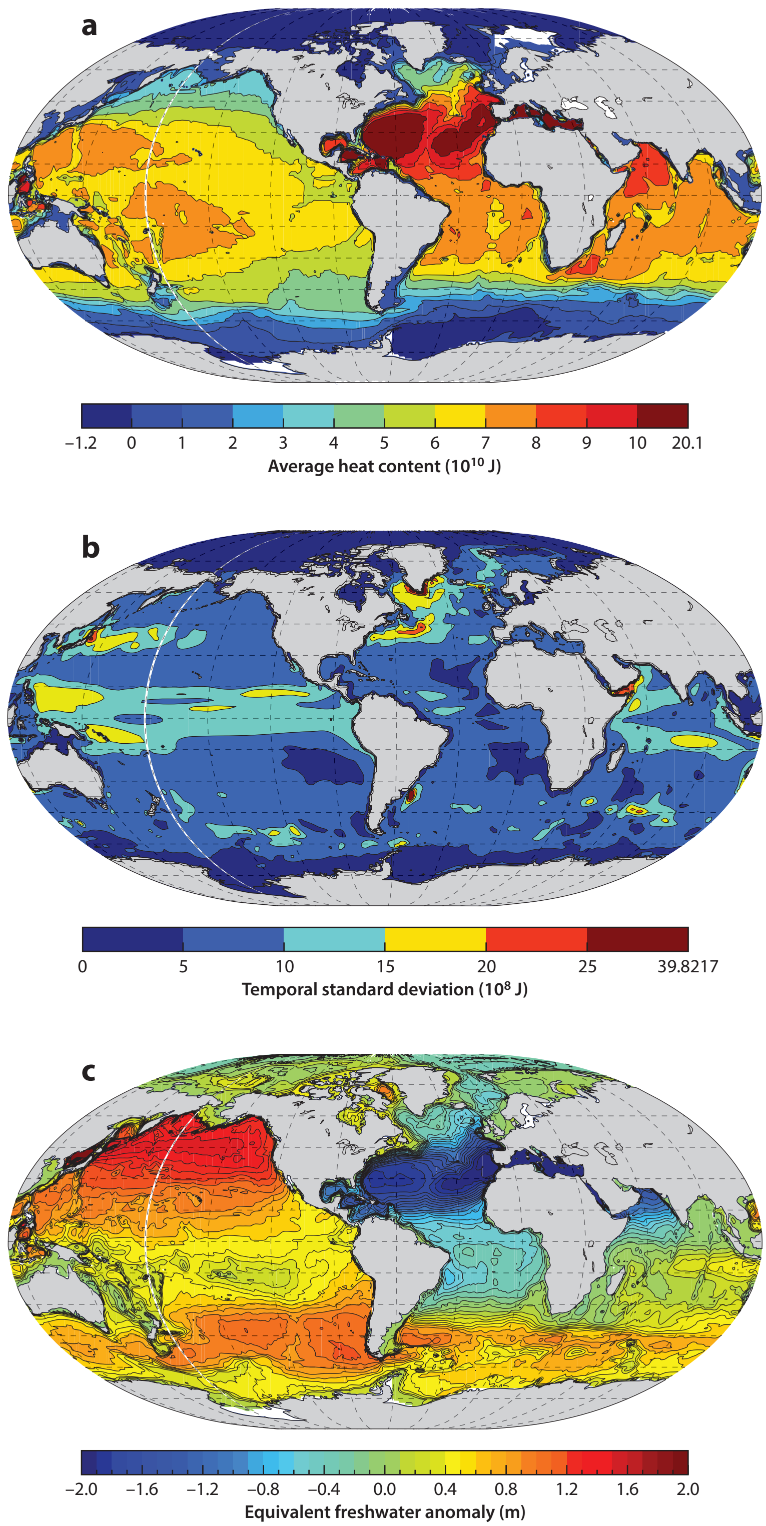
Figure 3

Estimated power-law exponents, q, from the altimetric frequency spectra, s^{-q}, showing the great spatial variability in these values. Combining the fields in **Figures 1–3** into a unified specification of background variability in frequency and wavenumber or in space-time covariances remains as a challenge. Data are from Hughes & Williams (2010) and C. Hughes (personal communication).

Figures 5 and **6** show the distribution of temperature samples in decadal intervals from the *World Ocean Atlas* (Locarnini et al. 2013; cf. Atkinson et al. 2014). A calculation of the corresponding mean temperature in, e.g., the 1930s must account for two visual features: the relatively heavy sampling of the Northern Hemisphere and, within that sample, the focus on the North Atlantic, a warmer-than-average region (**Figure 4*a***). Field expeditions often avoid high-latitude winter work, thus leading to a seasonal space-time alias with an expected warm bias. Hydrographic measurements to the seafloor did not become routine until the WOCE (World Ocean Circulation Experiment) era of the 1990s, and the volume surveyed in earlier decades is strongly regionally confined, although varying between decades. Oceanographic fashion often also dictates a focus

Figure 4

(*a*) Vertically integrated 20-year average heat content (a climatology) from an ECCO (Estimating the Circulation and Climate of the Ocean) version 4 state estimate. The very warm Atlantic and comparatively cool Pacific are among the most obvious features. Relatively dense sampling in one basin can bias the calculated average, and subsampling this field to produce a usefully accurate global average is challenging. Note that the zero line denotes an equivalent temperature of 0°C and not the absence of heat content in an absolute sense. $\overline{H}(\lambda, \phi)$ (where λ and ϕ are the longitude and latitude, respectively) has a strongly bimodal distribution, far from Gaussian (not shown). Modified from Wunsch & Heimbach (2014); see also Forget et al. (2015). (*b*) Temporal standard deviation of $H(\lambda, \phi, t)$. (*c*) The 20-year average salinity distribution in the ocean from the same state estimate, but converted into an equivalent freshwater anomaly. The conversion is relative to an ocean of depth 3,800 m and constant salinity $S_0 = 34.8$ g/kg. The comparatively salty Atlantic shows an equivalent deficit of freshwater relative to the fresher remainder of the world ocean.


a
−1.2
0
1
2
3
4
5
6
7
8
9
10
20.1
Average heat content (10^{10} J)
b
0
5
10
15
20
25
39.8217
Temporal standard deviation (10^{8} J)
c
−2.0
−1.6
−1.2
−0.8
−0.4
0.0
0.4
0.8
1.2
1.6
2.0
Equivalent freshwater anomaly (m)

on particular regions. Much of the history of physical oceanography can be inferred from the shipboard patterns visible in the coverage plots.

Lyman & Johnson (2014) and others have experimented with formulas for infilling regions of missing data (and see Schneider 2001). Expressions include the equivalent of simply assuming that values in the missing areas are equal to the spatial average of the samples; using nearest-neighbor interpolation; using objective mapping techniques with approximated noise and signal covariances **R** and **S**, respectively; and employing empirical orthogonal functions (singular vectors), a method for using **R** and **S**. Interpolation methods for infilling can reinforce the values that do exist. Thus, for example, if a majority of temperature values come from the relatively warm North Atlantic, extrapolating those values to unsampled regions further weights the warmest region. What is the best estimate to make of the mean? If some form of Equations 3 and 4 are to be used, the variance must be known (**Figure 4*b***). Because none of the various methods have been demonstrated to provide more accurate or precise averages than any other, what follows here uses only direct area- or volume-weighted averages of the samples that do exist, without any noise weighting, as a way of inferring the magnitude of the errors that must be managed.

Consider the calculation of the areal mean of $\overline{H}(\lambda, \phi)$ from the sampling distributions of different decades under the simplifying, if unrealistic, assumptions that the observations shown in **Figures 5** and **6** all reached the seafloor, were devoid of any systematic or bias errors, and had a uniform background noise. **Figure 7** illustrates the results of calculations based on these extremely optimistic and demonstrably incorrect assumptions, which, unsurprisingly, show a consistent warm bias. No implication is intended that the bias in published estimates from these intervals is as large as that shown in the figure—most authors attempted some compensation. For example, Montgomery (1958), when computing the Atlantic mean temperatures, discarded most of the available data in favor of a quasi-uniform coverage. The price was the combination of (*a*) data ranging in dates from 1901 to 1955 with no way to account for any temporal signal and (*b*) the violation of the statistical dictum that a good method should not result in the exclusion of relevant observations. Nonetheless, **Figure 7** shows the magnitude of the systematic error, of tenths of a degree Celsius. In terms of the straw-man accuracy above of 0.1 W/m^2, this bias error must be corrected on a decadal basis to the 1% level or better (**Table 1**). Durack et al. (2014) and Cheng & Zhu (2015), emphasizing the Southern Hemisphere gaps, concluded that spatial bias errors have led other investigators to substantially overestimate past oceanic temperatures and hence underestimate recent warming.

A different version of problem B is raised by **Figure 8**, which shows Stammer et al.'s (2013) estimate of the trend in sea level as determined from altimetric coverage, which is nearly global after 1992. The spatial average, $\langle \eta(t) \rangle$, of sea level relative to the geoid at some time t is not particularly interesting, as the computation and definition of the geoid are themselves somewhat arbitrary and difficult, and the dynamical equations governing the circulation depend only on the horizontal gradients, $\nabla_h \eta(\lambda, \phi, t)$. The main interest in global values of $\eta(\lambda, \phi, t)$ lies with the temporal change (the trends), because of its definition as sea level change, and with the connections through volume constraints with the heating and freshwater exchange rates. **Figure 9** shows the distribution of tide gauges, which provide the only available instrument measurements prior to the altimetric period. If the trend were anticipated to be spatially uniform, the issue reduces to a classical estimation problem with spatially varying noise. Here, physics precludes any uniform signal hypothesis.

3.3. Climatologies

The definition of an oceanic climatology is vague, but in practice it is simply a time average (as in **Figure 4**), $\overline{C}_b(\mathbf{r})$, over a period believed to be adequate to suppress irrelevant short-term temporal

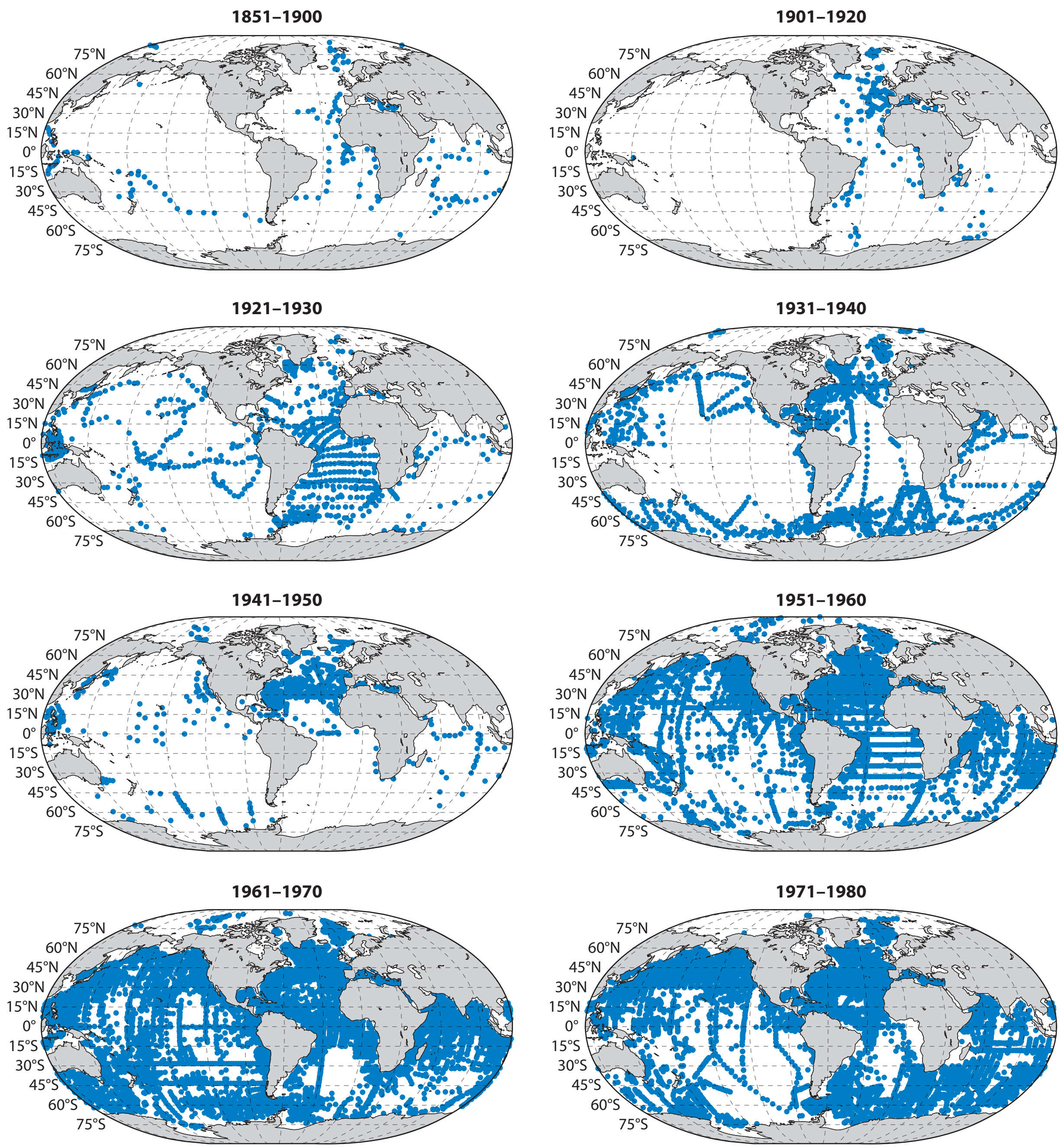


Figure 5

Measurements reaching at least 2,000 m for different time periods from 1851 and 1980. These and related plots must be interpreted in the context of determining the spatial averages of **Figures 4*a*** and **8**.

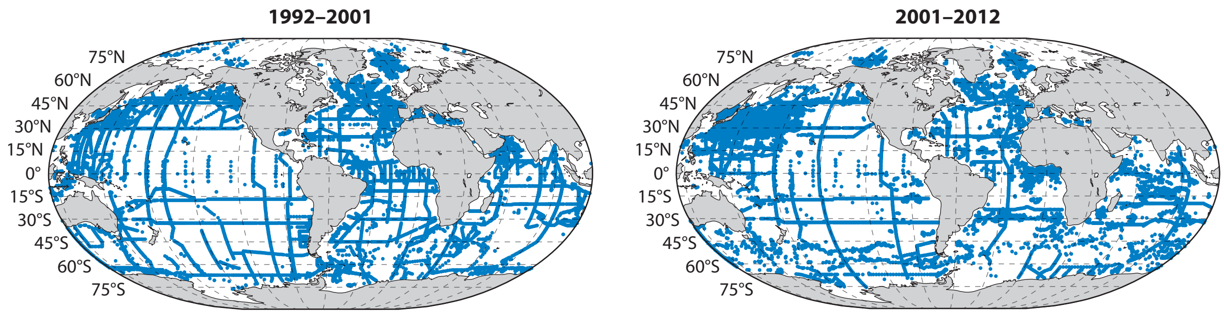


Figure 6

Locations of shipboard temperature and salinity data available during the two decades of the ECCO (Estimating the Circulation and Climate of the Ocean) version 4 state estimate, where the record extended to at least 2,000 m and usually to the bottom. Argo profiles (upper 1,000 or 2,000 m) available after around 2001 are not shown, nor is the large amount of nonhydrographic data used.

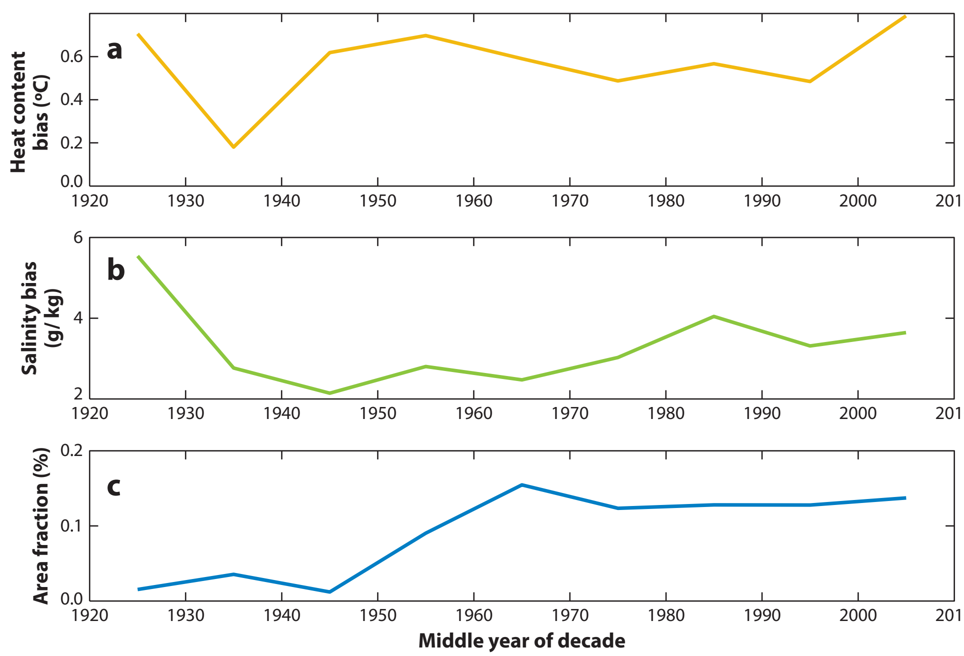


Figure 7

(*a*) Calculated bias in heat content computed by direct averaging of volume-weighted values from observation positions, converted to a mean temperature, as a function of decade. Values are of order 10% of the estimated mean temperature of 3.5°C. Any scheme used for correcting these biases must be accurate to at least the 1% level. (*b*) Salinity bias from spatial averaging of the same data positions as in panel *a*. As with temperature, the bias is nominally approximately 10% of the mean. Sensible averaging would again attempt to account for these biases, but their magnitude is daunting compared with the changes expected from, e.g., land-ice melt. (*c*) Percentage of 1° × 1° grid points covered in the model domain.

change. In the calculation of time change, taken up below, climatologies are often used to define anomalies in fixed time intervals, with the intent of both dynamically linearizing the apparent change and suppressing systematic errors believed to be fixed through time. Multidecadal global hydrographic climatologies (temperature and salinity) include those by Gouretski & Koltermann (2004) and Locarnini et al. (2013) as well as numerous regional versions.

Using anomaly values can also reduce certain kinds of measurement error. The most conspicuous such example is in variables that have a strong seasonal cycle, where the potential aliases are serious. If $C(\mathbf{r},t) = C_0(\mathbf{r},t) + a_0(\mathbf{r})\cos(2\pi t/T_a - \phi_a)$ and $T_a = 1$ year, other noise sources are omitted, and measurements are made at irregular times t_i, then the average,

$$\tilde{m} = \sum_{i=1}^{M} a_i [C_0(\mathbf{r}, t_i) + a_0(\mathbf{r})\cos(2\pi t_i/T_a - \phi_a)], \tag{9}$$

will alias into lower frequencies, including $\tilde{m}$ at the apparent zero frequency (Bracewell 1986). For some variables, such as sea surface temperature, the annual cycle and its harmonics may have 90% or more of the record variance, and its near-perfect removal before averaging becomes very important. The simple expedient of first subtracting a climatological annual cycle can often remove nearly all of the alias, thus greatly reducing the potential error. One new challenge then is determining the changes in that component over years and decades, and little is known about how to solve this problem. (For a review of the annual cycle, see Vinogradov et al. 2008.)

The hope is that the use of a climatology can make the precision of a time change much more accurate. In terms of the temporal change that is the present focus, and relative to a nonseasonal climatology,

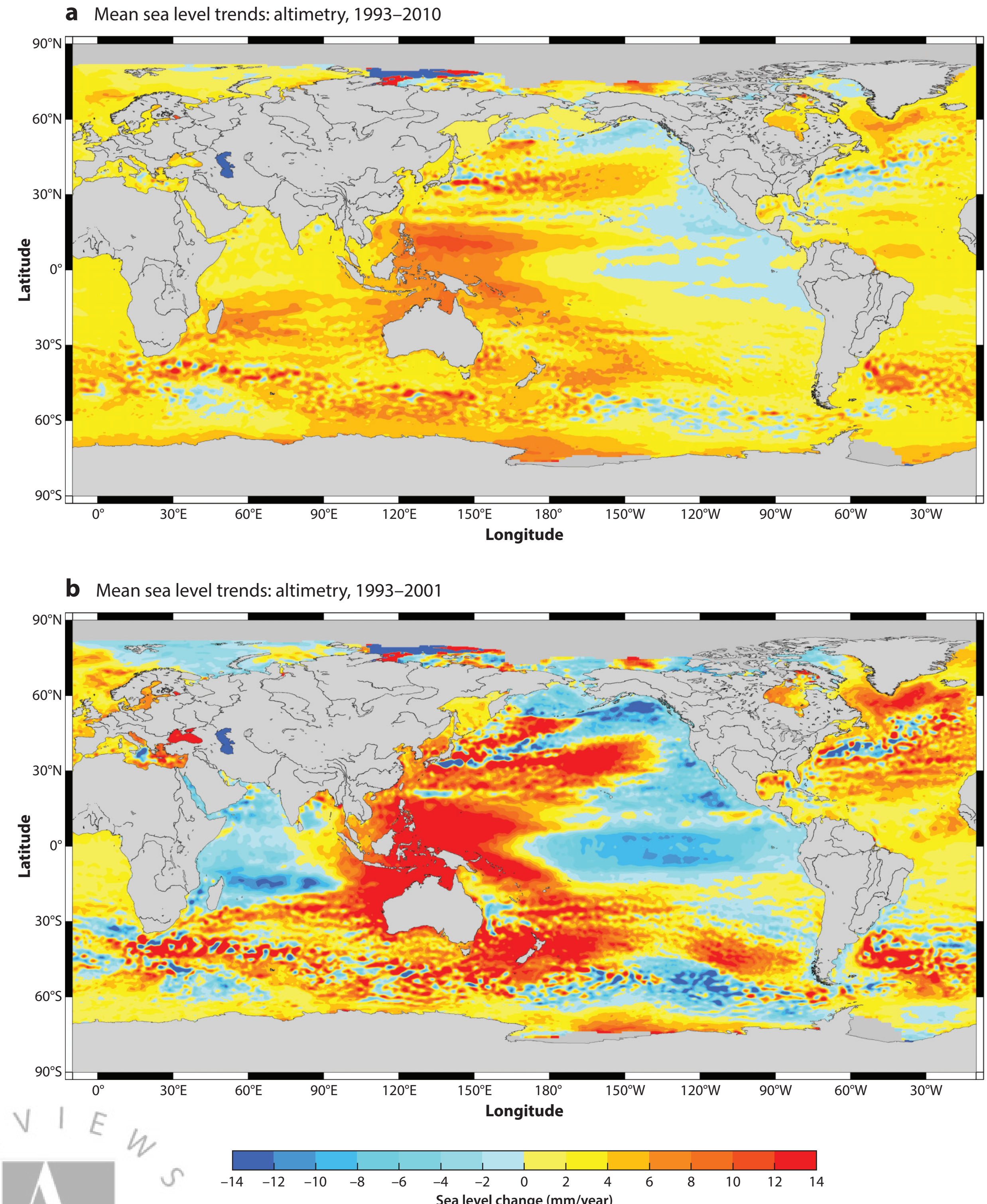
$$\Delta C(\mathbf{r}, t_{\mathrm{F}}, t_{\mathrm{I}}) = [C(\mathbf{r}, t_{\mathrm{F}}) - \overline{C}_b(\mathbf{r})] - [C(\mathbf{r}, t_{\mathrm{I}}) - \overline{C}_b(\mathbf{r})] \tag{10}$$

appears to be independent of $\overline{C}_b$. Lyman & Johnson (2014) and Cheng & Zhu (2015) concluded that during data-sparse times, results were qualitatively dependent on the choice of $\overline{C}_b(\mathbf{r})$. As elsewhere, details matter: Interpolation is being carried out from one time-average grid, of varying accuracy and reliability, to another grid, and interpolation error inevitably intrudes. Typically, interpolation would be done from the denser grid (often the climatology) to the less dense (often the newly available observations). The trap of assuming that because a field has been uniformly gridded it is thus uniformly reliable has not always been avoided (see, e.g., the extensive literature using the AVISO gridded altimetry, which is pleasingly smooth and easy to use[7]). Roemmich et al. (2012) estimated the temperature change from the RRS *Challenger* era in the 1870s to the Argo era for the region above 900 m. They interpolated the more dense Argo values to the positions of the *Challenger* data set and also accounted for the radical technology change. The bias from estimating the change only at the locations measured in the 1870s can at least be bounded; a similar calculation over the full water column is not now possible. With or without a reliable climatology, changes taking place beyond the observed region require either suppression (values assumed to be zero there) or extrapolation, with all of the associated pitfalls.

4. TIME CHANGES AND TRENDS

Apparent regional and global trends in oceanic properties are the subject of an ever-expanding literature. The central physical conundrum is perhaps best described via a thought experiment:

[7]Heavy space-time averaging, extrapolation, and known spatial variations in the measurement errors of many types mean that considerable care is required when interpreting and using this type of product (Ducet et al. 2000).


a Mean sea level trends: altimetry, 1993–2010
90°N
60°N
30°N
0°
30°S
60°S
90°S
Latitude
0°
30°E
60°E
90°E
120°E
150°E
180°
150°W
120°W
90°W
60°W
30°W
Longitude
b Mean sea level trends: altimetry, 1993–2001
–14 –12 –10 –8 –6 –4 –2 0 2 4 6 8 10 12 14
Sea level change (mm/year)

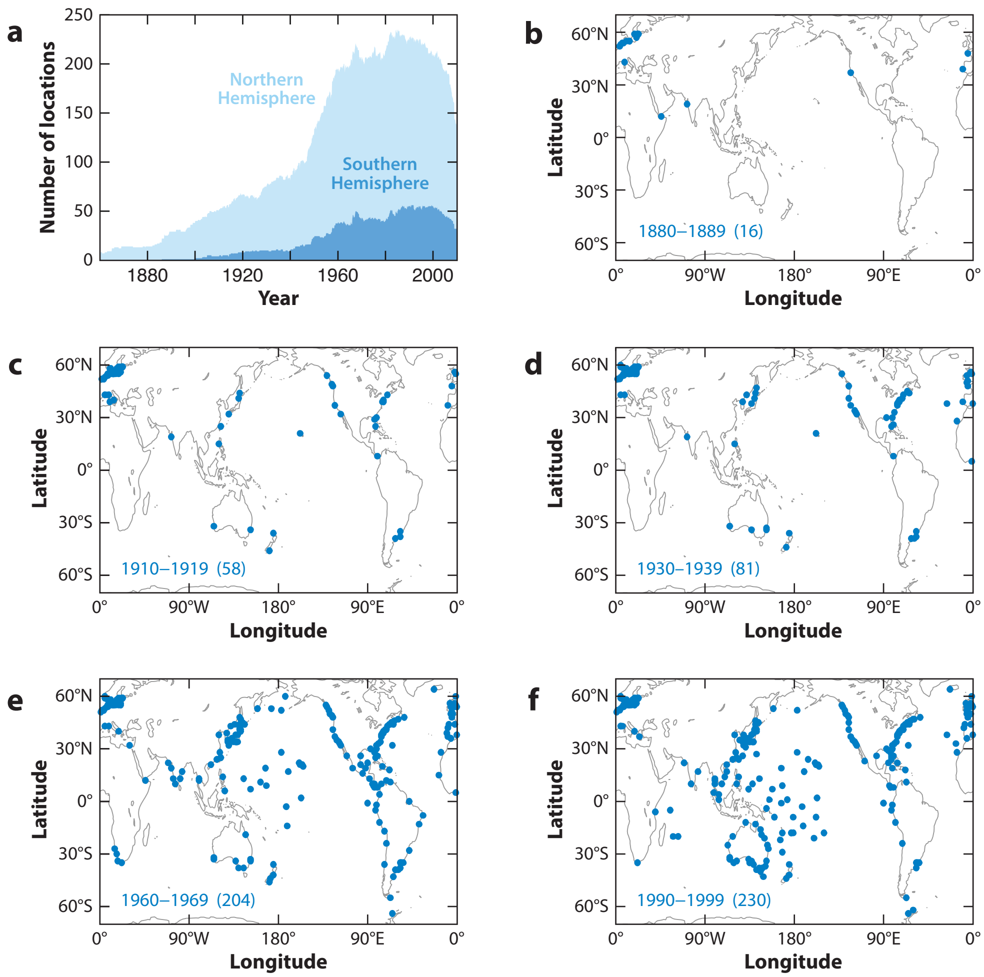


Figure 9

(*a*) Number of Northern and Southern Hemisphere tide gauge data sets available for estimating global sea level trends, as used by Church & White (2011). (*b–f*) Tide gauge positions during different time periods from 1880 to 1999, with the number of positions given in parentheses. The spatial sampling problem is evident.

Consider an equilibrium Earth that has a fully coupled ocean, atmosphere, cryosphere, biosphere, and lithosphere but is not subject to any external disturbances beyond the seasonal solar forcing (no solar or orbital variability or imposed anthropogenic gas emissions). This system state would vary only from internal (endogenous) processes such as land-ice dynamics, sea-ice changes, volcanic eruptions, weather, ocean eddies, and the like. The turbulent atmosphere and

Figure 8

Trend in sea level as determined from altimetry alone over the intervals (*a*) 1993–2010 and (*b*) 1993–2001. Note that the highest latitudes are not included. Values are spatially complex from a combination of thermal expansion/contraction, addition/removal of freshwater, equation-of-state nonlinearities (Gille 2004, Schanze & Schmitt 2013), wind forcing, and ocean volume change effects. Reproduced from Stammer et al. (2013).

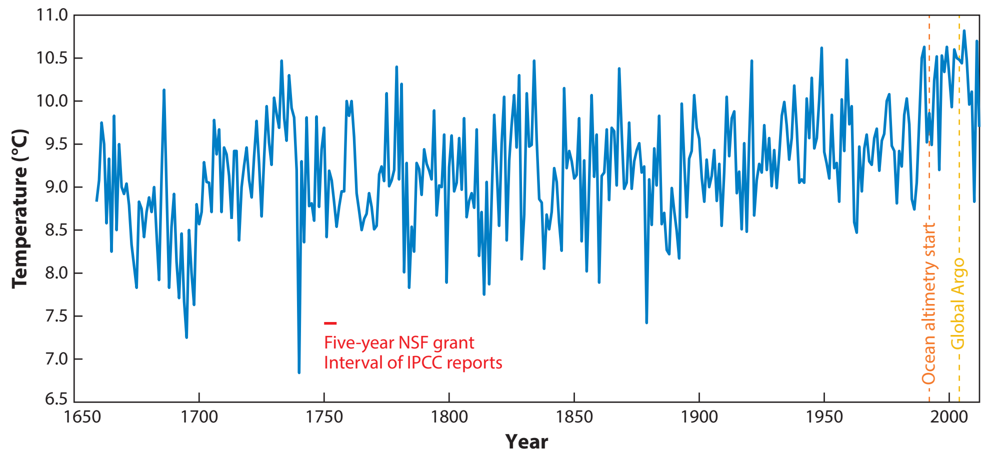


Figure 10

The central England temperature record, showing multidecadal trend-like behavior. Determining whether the apparent trend across the entire record length is due to external forcing or internal fluctuations involving the timescales in **Supplemental Figure 1** is the crux of the climate trend problem. (Representation on a Kelvin scale—adding 272.15°C—would be a useful reminder of the difficulties of determining slight changes in the near-equilibrium system.) Data are from Manley (1974) and the UK Meteorological Office website (**http://www.metoffice.gov.uk**).

ocean will be exchanging heat, freshwater, carbon, etc., on all timescales, from those of weather to any deriving from, e.g., long-term instabilities of continental land ice (see **Supplemental Figure 1**; follow the **Supplemental Materials link** from the Annual Reviews home page at **http://www.annualreviews.org**). Heat, freshwater, carbon, etc., will flux among the elements of the system and be stored for varying times in different parts of the system. The least likely behavior of such a system is an unchanging partition among the elements of the storage of this heat, freshwater, carbon, etc. In this thought experiment, a plausible outcome of long integrations is a random-walk-like migration of anomalies in these components, of both signs, between the ocean, atmosphere, cryosphere, biosphere, and lithosphere. Linear or other apparent trends in the ocean component can corrupt a real trend, with either sign.

Hasselmann (1976) described the counterintuitive behavior of oceans subject to stochastically integrated forcing [compare the probabilist's classical game of Peter and Paul, which involves the expected winnings through time from repeated coin flips (Feller 1957)]. The Hasselmann model is the simplest case of a wide class of random processes that are able to describe many, if not most, observed climate time series. One general form is the ARMA(L,M) (autoregressive moving average) process,

$$\begin{aligned} &y[(n+1)\Delta t] - \alpha_1 y(n\Delta t) - \cdots - \alpha_L y[(n-L-1)\Delta t] \\ &= \theta(n\Delta t) + \gamma_1\theta[(n-1)\Delta t] + \cdots + \gamma_M\theta[(n-M)\Delta t], \end{aligned} \tag{11}$$

where $\theta(n\Delta t)$ is stationary zero-mean white noise with variance σ_θ^2. Depending on the constants, α_i and γ_j, such processes produce apparent trends over arbitrarily long durations, including those of the record length (see **Figure 10**). [Hasselmann's model was an ARMA(1,0), i.e., an AR(1); among other examples, Hughes & Williams (2010) (**Figure 3**) represented sea level change as a spatially varying autoregression, mainly of orders 3–6.]

Descriptions such as Equation 11 are easily obtained from the solutions to general linear partial differential systems in space and time and are readily transformed to or from general state space forms (Brogan 1991).[8] The stationarity and spectral shape (power laws, peaks, and plateaus) of the system are controlled by the positions of the zeros of the polynomials

$$1 - \alpha_1 z - \cdots - \alpha_L z^L, \quad 1 + \gamma_1 z + \gamma_2 z^2 + \cdots + \gamma_M z^M \tag{12}$$

(see, e.g., Priestley 1982, Box et al. 1994). These expressions are readily generalized to nonstationary (e.g., an autoregressive integrated moving average) and nonlinear forms (Seber & Wild 1989) of an infinite variety, including so-called long-memory processes (Beran 1994, Percival et al. 2001), discontinuous elements, etc. Purely statistical means cannot distinguish whether an observed time series has the form of Equation 11 or any other stochastic representation, has a linear trend component

$$x[(n+1)\Delta t] = a + [(n+1)\Delta t]b + y[(n+1)\Delta t] \tag{13}$$

(where a and b are constants), or has more complex trend forms than a straight line (see, e.g., Dangendorf et al. 2014, Jordà 2014, Ocaña et al. 2015, Visser et al. 2015). Apparent trends appearing in stochastic $y(n)$ can have either sign.

The ubiquity of integrating processes in the ocean implies that apparent trends are always expected, and a compelling null hypothesis is that these trends are the result of an internal exchange within the climate system, to be rejected only with strong evidence.[9] To proceed, very strong, explicit, a priori physical hypotheses are required. A full Bayesian approach would be sensible, with a specified trend being given an appropriate a priori probability—which would be both specific and subject to debate.

Consider the central England temperature record $T(m\Delta t)$ (Manley 1974) (**Figure 10**), whose behavior is at least partially stochastic (e.g., Tung & Zhou 2013). Multidecadal trends of both signs are apparent, as is a slow rise across the entire record length. Is that slow rise of anthropogenic cause, or is it an example of a stochastic summation of the complex of interchanges of the ocean and atmosphere?

Stochastic elements appear in all forces driving the ocean, with the exception of the tides. Many authors have discussed the responses of sometimes very complicated models to those aleatory elements (e.g., Stephenson et al. 2000, Cessi & Louazel 2001, Thomas & Zhai 2013).

5. GLOBAL-SCALE INFERENCES: INTERPRETATION

5.1. Heat/Temperature

Oceanic temperature has attracted more attention than any other variable, both because of its fundamental nature in ocean physics and, more recently, as the result of intense public attention to the question of whether there has been a true "hiatus" in global warming. The default explanation of an apparent slowing of atmospheric global average surface temperatures is that the supposed "missing" heat from the greenhouse gas increase has entered the ocean. Many competing calculations exist for differing time intervals, depth ranges, instrumental types, and statistical hypotheses, including methods for removing the warm bias.

[8]Discrete representations avoid the mathematical complexities of defining white noise and other processes, which require great care in continuous time and space (see, e.g., Gardiner 2004).

[9]With the urgent problem of global warming, scientifically reasonable statements that perceived trends cannot be distinguished from background variability have sometimes led to public ad hominem abuse.

5.1.1. Background values. Oceanographers measure temperatures, and the integrated value is the heat content. As used in **Table 1**, an approximate conversion between heat content and temperature is $\langle \overline{H} \rangle = \rho c_{\mathrm{p}} \langle \overline{T} \rangle V$, where the nominal mean ocean density is $\rho = 1,038$ kg/m^3, the heat capacity is $c_{\mathrm{p}} = 4{,}000$ J/(kg $\cdot$°C), and the volume is $V = 1.3 \times 10^{18}$ m^3. In practice, $\langle \overline{H} \rangle$ is computed by volume and time integration of $T(\lambda, \phi, z, t)$ with a suitably varying ρc_{p}.

5.1.2. In situ measurements. The first attempts at estimating the global mean temperature from in situ measurements were probably those of Krümmel (1907) and Montgomery (1958) (see discussion in Worthington 1981). Montgomery (1958) estimated the mean oceanic temperature as 3.52°C, and Worthington (1981) estimated it as 3.51°C (all potential temperatures), with no uncertainty estimates provided in either case. No dates were assigned, as the entire historical record was used.

Instrumental accuracies. Over an interval on the order of a decade, instrumental technologies change little, and if a homogeneous set of instruments was used ten years apart, the discussion could become one of instrumental precision rather than accuracy. Over multiple decades, and definitely over a century, technologies change radically, and the absolute accuracies must be known. For the expendable bathythermograph era, a considerable effort has been directed at inferring the systematic errors in what might have appeared to be a homogeneous measurement system, with some bias estimates of order 0.2°C between instrumental batches, but these biases are both time and depth dependent (Abraham et al. 2013).[10]

Sampling distribution changes. A hypothesis of uniform distribution of heat content change can be definitively ruled out on the basis of estimates of net surface exchange showing major regions of heating and cooling; the measured behavior of passive tracers; and the regional physics of convection, Ekman pumping, upwelling regions, etc. Many estimates (e.g., Ishii & Kimoto 2009) are made by assuming that thermal shifts are confined to the upper ocean alone, where "upper" commonly means 700 m or sometimes 1,000 m. But the hypothesis must break down on the timescales over which the deeper ocean begins to respond (**Supplemental Figure 1**): The problem is a strong function of duration. Passive tracers, to the extent that they are analogues of the active thermal changes, show surface-injected fluorocarbons, tritium, and other tracers on the oceanic floor within one or two decades of injection, over small but nonnegligible regions that grow with time (e.g., Östlund & Rooth 1990; Smith et al. 2005; X. Liang & C. Wunsch, manuscript in review). Another clue is the highly diluted carbon-14 content observed in the near-surface ocean, with values[11] well below atmospheric equilibrium values. The difference is consistent with a massive continuing exchange with the low-radiocarbon reservoir below. Adjustment times in the ocean circulation are a near continuum of values and physics ranging from seconds to a full equilibrium time beyond 10,000 years (see figure 11.15 of Wunsch 2015 or **Supplemental Figure 1**).

The recent Argo float coverage makes a qualitative change in what is possible in estimates of integrals and averages. As an example, **Figure 11** shows the result of sampling **Figure 4*a*** using a uniform random distribution of positions. The bias error is smaller than the variance, which is much reduced once the number of independent samples exceeds approximately 12,000 but which

[10]Most of the bias arises from the instrument depth estimate rather than from the thermal sensors.

[11]Values are usually reported as the logarithm of the concentration. These are then divided by the decay constant (Bard et al. 1994), producing an apparent "age" that must be interpreted warily.

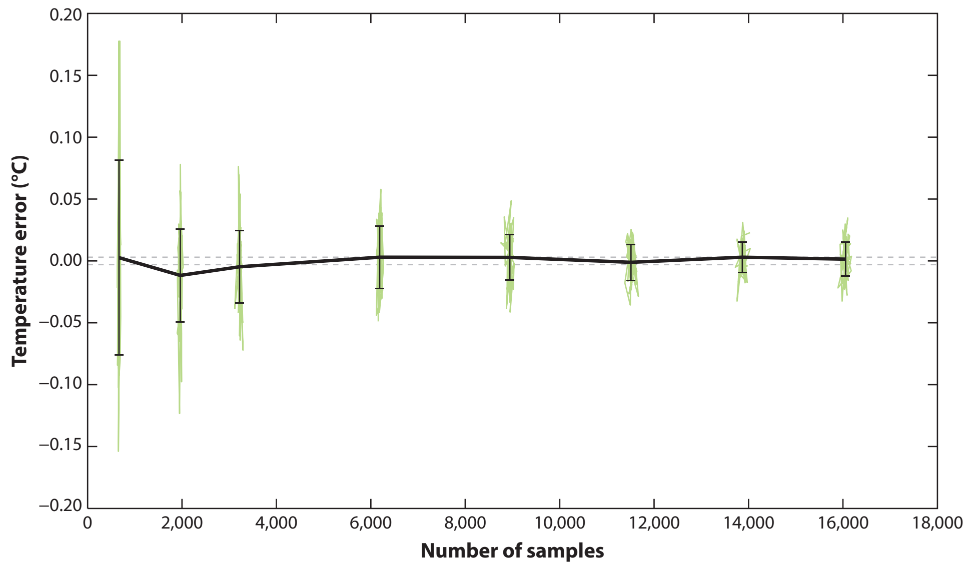


Figure 11

Temperature error in the calculated mean from a randomly sampled $\overline{H}(\lambda, \phi)$ with up to 16,000 samples, each done 50 times. With 16,000 samples, approximately one-third of the ocean area is represented. The error bars show the standard deviation from each of the 50 sets, and the horizontal dashed lines indicate an accuracy goal of 0.003°C based on a net heating error of 0.015 W/m^2 per decade.

remains nonnegligible. These values assume that most of the profiles have uncorrelated noise.[12] Experiments (not shown here) with random sampling of the eddy-resolving/permitting ECCO2 (Estimating the Circulation and Climate of the Ocean, Phase II) state estimate (Menemenlis et al. 2005) are consistent with reduction of the squared bias error below the variance, with homogeneous coverage of approximately 10% of the ocean volume (see also Ponte 2012).

With in situ data alone, the Argo era is sensibly handled as a special case (Roemmich et al. 2015), approaching a homogeneous spatial random sample. Three thousand floats diving to 1 or 2 km produce approximately 9,000 profiles per month or approximately 10^5 per year. Roemmich et al. (2015) estimated the heating rate to 2,000 m as 0.4–0.6 W/m^2, a possible temperature range (not a standard error). The effect of the omitted deep ocean is likely less than a 10% contribution (Wunsch & Heimbach 2014), but one expected to grow as the Argo period extends to multiple decades, eventually encompassing most of the signal: Nearly half of the ocean volume lies below 2,000 m (see discussion of the ocean long-memory problem in Wunsch & Heimbach 2014). Other missing regions include marginal seas, continental shelves, the Arctic, and subsea-ice areas generally.

5.1.3. Inputs and outputs. Balancing a budget requires consistency of temporal mean changes with the net heat inputs and outputs, and determining these is even less straightforward than determining the interior change. Historically, air-sea transfers were computed using bulk formulas involving empirical drag and exchange coefficients, estimated air-sea temperature differences, and

[12]A concern, particularly in regions of strong eddy variability, is that individual floats can remain trapped within particular eddies over many months—thus producing a temporally and spatially correlated noise. The effect does not appear to have been evaluated (D. Roemmich, personal communication).

wind stress (reviewed by Josey et al. 2013). Integrating these transfers globally results in residuals of several tens of watts per square meter.

Numerous reports have reviewed the estimation of the radiative forcing of the Earth (e.g., Myhre et al. 2014). The measurements and calculations are difficult, involving the incoming solar radiation, cloud cover, aerosols, and instrumental problems. For example, Stephens et al. (2012) inferred that satellite measurements produce a top-of-the-atmosphere global imbalance uncertainty of ± 0.4 W/m^2 by using oceanic estimates of heat uptake (thought to remove bias errors of over 2 W/m^2). The result could be accurate at this level, but in such circumstances, the calculated ocean heat uptake is no longer independent of the estimated forcing, and closure of the heat budget becomes circular.

5.2. Salinity/Freshwater

The problem of changing oceanic freshwater and salt content is analogous to that for temperature but is considerably more difficult with many fewer data, and so the discussion here is brief (see Skliris et al. 2014). **Figure 7** shows the bias error for the mean salinity, computed from the field in **Figure 4*c***; this error is analogous to that computed for temperature and is, again, very roughly 10% of the mean value, with a corresponding need to greatly reduce it to obtain the accuracy (or precision) of the sea level equivalent of 0.1 mm/year.

The ocean represents the major freshwater reservoir of the global hydrologic system, with transfers in and out being a minute fraction of the reservoir total, so determining whether the volume of freshwater has been changing is a formidable problem in data accuracy. As with all physical processes, the two possible approaches involve determining the net difference of sources and sinks integrated over decades and direct measurements of the changing freshwater volume. Neither is easy.

Background, quasi-steady net precipitation over the ocean is of order 12 Sv, river runoff has been estimated as 1.2 Sv, and percolation through the continents directly into the ocean is 0.07 Sv. Runoff and percolation include contributions from melting land ice, estimated as 0.07 Sv for Antarctica and 0.005 Sv for Greenland (for references, see table 3.1 of Wunsch 2015 or the online **Supplemental Material**), all with large uncertainties. The major sink of oceanic freshwater is evaporation, which is difficult to measure (Yu 2007) and which is often estimated by assuming an oceanic steady state such that the annual mean net evaporation is, within a very small margin of error, by definition equal to the net input.

A 1% change in the precipitation rate, approximately ± 0.1 Sv, competes with and dwarfs the estimated changes in incremental melting ice inputs. If evaporation can be assumed to balance precipitation to an accuracy exceeding ± 0.01 Sv on an interannual basis, then the problem of determining net evaporation minus precipitation is solved at a useful level. Such an accurate balance of very different physical processes seems implausible but cannot be ruled out on the basis of existing observations.

Direct inferences of changes in oceanic freshwater storage are based on measurements of salt content. Although definitions have changed technically over the years, it suffices (and is now once again acceptable practice; see Millero et al. 2008, McDougall et al. 2012) to treat salinity, S, as the relative fraction of salt in grams per kilogram of seawater. One of the earliest calculations of mean salinity was made by Montgomery (1958), who estimated it at 34.71 g/kg; Worthington (1981) later estimated it at 34.72 g/kg (no uncertainties provided). Calculations underlying **Figure 4*c*** give a mean of approximately 34.8 g/kg (Wunsch & Heimbach 2014). These numbers are not easily understood intuitively but are convertible using Equation 1. Changing sea level by 1 mm/year globally corresponds to a volume input of approximately 0.01 Sv or $\langle \Delta S \rangle \approx 3 \times 10^{-6}$ g/kg per decade—the accuracy required to discuss inputs and outputs. Wang et al. (2013) reported Argo salinity biases, $\mathcal{E}(\Delta S)$, that sometimes exceeded 0.05 g/kg. Antonov et al. (2002) have estimated the net salinity change (see also the discussion in Wadhams & Munk 2004 and Durack & Wijffels 2010).

5.3. Sea Level

Determining and understanding changes in global mean sea level brings together almost every element of physical oceanography, including all of those involved in understanding temperature and salinity changes as well as many others. Thermal expansion and contraction, freshwater addition and removal, and wind stress changes, among other phenomena, change sea level regionally and globally. The magnitudes of the spatial variations are directly connected to the energy budgeting problem.

Historically, sea level change was the domain of regional geodesy and engineering for beach erosion, power-plant siting, flood control, etc. It became of widespread public and scientific interest with anxiety about global rise along with greenhouse gas forcing. Until around 1990, the only instrumental data were obtained from tide gauges (**Figure 9**). Many calculations are based on the use of singular vectors (empirical orthogonal functions) that were assumed to be representative. Some serious statistical issues arise in their use, but discussion space is lacking here; for a summary and references, see the online **Supplemental Material**.

Numerous reviews have addressed the global sea level change problem over a variety of time intervals (for references, see Church & White 2011, Stammer et al. 2013, or any of the IPCC reports). Because thermal expansion/contraction and freshwater addition/removal are the two dominant contributors to global mean sea level on nongeological timescales, attempts to close the sea level budget at a useful level of accuracy encounter all of the same problems as calculations of heat and freshwater uptake. Accuracy-precision discussions of changes in the altimeter-Argo period are, again, very different from those for the preceding decades.

5.4. Energy

Global oceanic energy budgets represent dynamics in a closer connection than do those of heat or salt, and thus the associated discussions have a somewhat different flavor. A considerable body of literature—one too large to review here—has attempted to delineate and balance the oceanic energy budget (see the textbook discussions in chap. 13 of Thorpe 2005 and chap. 3 of Huang 2010, as well as, e.g., Ferrari & Wunsch 2009 and Eden et al. 2014). In some cases (Winters & Young 2009, Huang 2010, Roquet 2013, Tailleux 2013), questions linger even about the definition of some of the variables, especially those connected to potential energy and its division into available and unavailable components. Considerable complexity arises from the nonlinear equation of state of seawater. Owing to space limitations, a bit more discussion of oceanic energy has been placed in the online **Supplemental Material**.

At the present time, a sketchy summary might be that the dominant energy inputs come from the time-mean wind stress and its fluctuations (particularly in the inertial frequency band), surface buoyancy forcing, and geothermal heating (0.1 W/m^2, approximately 37 TW; see Davies 2013). In an ocean without surface forcing (Ashkenazy et al. 2014), a significant general circulation can result (order 30 Sv) from the latter, but the conversion of thermal forcing to kinetic energy is relatively inefficient. (In thermodynamic equilibrium, the total geothermal flux, including the land contribution, would necessarily be part of what the atmosphere must radiate back to space; 0.1 W/m^2 is not small compared with the accuracy requirements.) More generally, the background energies of all types are so large compared with any physically plausible change that global shifts in total energies cannot now be detected, although regional changes have been discussed.

6. NATURAL INTEGRATION

Nature provides some notable integrals that have attracted much attention over the years. Interpreting them is an interesting problem.

6.1. Transport Choke Points

The ocean circulation transports scalar properties such as temperature (heat), salinity (freshwater and salt content), nutrients, carbon, and oxygen. Elements of the circulation provide choke points, which are natural integrations whose relative ease of monitoring has been attractive. The major examples are the Florida Current (the Gulf Stream; Schmitz & Richardson 1968, Meinen et al. 2010), the Indonesian Passages (Gordon et al. 2010), and the Drake Passage (Baker et al. 1977, Firing et al. 2014).

Consider the Florida Current at approximately 26°N, bounded on the west by Florida and on the east by the Bahama Banks, and with the lengthier remainder of the ocean at that latitude terminated at Africa. Along a zonal section, spanning both elements, the transport of any property is

$$\underbrace{\iint_{\text{Florida Straits}} \rho C v_{\mathrm{n}}(\mathbf{r}, t)\mathrm{d}A(\mathbf{r})}_{V_1} + \underbrace{\iint_{\text{Bahamas to Africa}} \rho C v_{\mathrm{n}}(\mathbf{r}, t)\mathrm{d}A(\mathbf{r})}_{V_2} \approx 0, \tag{14}$$

where v_{n} is the flow normal to the section. (Exact balance is precluded by small storage terms, the Bering Sea inflow, and evaporation-precipitation.)

If V_1 alone is measured, can it be interpreted in terms of variations in V_2 and their causes? Reality intrudes in several ways. For example, (*a*) the interior circulation is a complex of various physics (Wunsch & Heimbach 2013, Gray & Riser 2014, Thomas et al. 2014), (*b*) much of the fluid in the observed Gulf Stream enters from the Southern Hemisphere and ultimately derives from everywhere (e.g., Schmitz & Richardson 1991), and (*c*) fluid entering the deep western boundary current from the north is rapidly recycled into the interior long before it moves very far latitudinally (e.g., Lozier 1997). The first issue has been addressed by deploying instruments across the entire ocean width at the Florida Straits latitude (through a program called RAPID; Rayner et al. 2011). The efficacy of this array is not in doubt, but the logistic appeal of the Florida Straits choke point has been lost by the necessity of instrumenting the entire ocean width.

A generic representation of transport, $V_{\mathbf{r}}$, with fluctuations between two horizontal points, $\mathbf{r}_1$ and $\mathbf{r}_2$, above depth z_0, in terms of Green functions, G, is

$$\begin{aligned} V_{\mathbf{r}}(\mathbf{r}_1, \mathbf{r}_2, z_0, t) \;=\; & \iiint_{\text{ocean boundaries, } t'<t} G_{\mathrm{B}}(z_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}', t, t') q_{\mathrm{B}}(\mathbf{r}', t')\mathrm{d}\mathbf{r}'\mathrm{d}t' \\ & + \iiiint_{\text{ocean volume,} t'<t} G_{\mathrm{I}}(z_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}', t, t', z') q_{\mathrm{I}}(\mathbf{r}', t')\mathrm{d}\mathbf{r}'\mathrm{d}z'\mathrm{d}t'. \end{aligned} \tag{15}$$

G has been divided into two parts: the boundary Green function, G_{B}, representing the transport disturbance caused by externally prescribed forcing (wind stress, air pressure fluctuations, and thermodynamic exchanges at the sea surface and seafloor), and the interior Green function, G_{I}, representing the transport disturbance caused by representable interior forcing, including fluctuations in the eddy field, either as computed or parameterized with a space-time statistical prescription. The variables q_{B} and q_{I} are the externally imposed disturbances propagating via G_{B} and G_{I}, respectively. Longuet-Higgins (1965) analyzed an example of G_{B} for transport in a one- and two-layer ocean on a β plane. Transport fluctuations caused by time-dependent deterministic forcing are the bread and butter of classical circulation theory and textbooks (e.g., Anderson

& Corry 1985). More generally, Green functions appropriate to this problem have rarely been computed explicitly but are implicit in general circulation model calculations.

Again, choke points are integrators, summing up disturbances that include those from long ago and far away, and they must exhibit random-walk-like behavior. The boundary conditions, $q_{\mathrm{B}}(\mathbf{r},t) = q_{\mathrm{BD}}(\mathbf{r},t) + q_{\mathrm{BS}}(\mathbf{r},t)$, have deterministic and stochastic components (q_{BD} and q_{BS}, respectively). Given q_{B} with error estimates, Equation 15 permits calculation of the resulting $V_{\mathbf{r}}$. When written in discrete form, a set of simultaneous equations for $q_{\mathrm{B,I}}(n\Delta\mathbf{r}', q\,\Delta t')$ results that involves the measurement error of $V_{\mathbf{r}}$. The skill of a solution to this linear inverse problem depends directly on the number and accuracy of the measurements of $V_{\mathbf{r}}$ and the reach of $G_{\mathrm{B,I}}$ in space and time. These considerations lead directly to the question of the distances and times over which earlier disturbances can lead to measurable fluctuations in $V_{\mathbf{r}}$ (Heimbach et al. 2011). Many authors—including Cessi & Louazel (2001), Cessi et al. (2004), and Stammer (2008)—have considered transport disturbances, most recently in the context of sea level shifts (a major part of the governing pressure field), but no general discussion exists. A strong inference is that no regional observation system, even one for entire zonal sections, will lead to oceanic understanding—only global data will do so.

6.2. Earth's Rotation and Polar Motion

Redistributions of mass in the ocean, atmosphere, and cryosphere affect both the position of the rotation pole (polar motion) and the length of day (spin rate). These are major subjects in their own right (e.g., Munk & MacDonald 1960, Lambeck 1980). In the present context, note the discussions by Munk (2002) and Mitrovica et al. (2006) of the difficulties of reconciling polar motion with estimates of land-ice melt and sea level change, which become part of any serious claims of having closed the dynamical and physical conservation budgets (see also Dickey et al. 2008, Nerem & Wahr 2011, Chen et al. 2013). Glacial isostatic adjustment (postglacial rebound) effects with their long timescales have to be incorporated into models and estimates of recent oceanic change.

6.3. Acoustic Integration

That acoustic travel times in the ocean represent (primarily) integrals of heat content is the basis of ocean acoustic tomography and the subject of another literature (Munk et al. 1995). Again because of space limitations, I note here only the discussions by Dushaw & Menemenlis (2014) and the possibilities of wider use of natural sound sources (e.g., Brown et al. 2014).

7. GOING FORWARD

7.1. Syntheses

Most physical components of oceanic change are interconnected. A measurement of a change in sea level height, η, must make physical and dynamical sense in terms of changing temperatures, salinities, air-sea exchanges, wind fields, sea-ice melt, Earth's rotation, etc. A statistical axiom is that all relevant data must be included to obtain a best estimate. Calculations of integrals or changes excluding all but one observation type are primarily useful analyses for understanding those data but are potentially irrelevant if the results conflict without explanation with other data types or known physics. That observed temporal changes in oceanic properties must obey the equations of motion and thermodynamics and be quantitatively consistent with atmospheric and cryospheric shifts is a general and powerful source of information and constraint.

These two ideas—that different physical variables are related and that they are bound together by the equations of fluid dynamics—have led to an oceanic synthesis effort referred to as state estimation. One such effort produced **Figure 4** as part of work reviewed by Wunsch & Heimbach (2013). State estimation can be regarded as a complete interpolation/extrapolation procedure based on the full dynamics of a general circulation model and all of the data. An important caveat is that dynamics, data, and a diverse set of a priori beliefs must be combined and used sensibly. As is the case with forward models, not all state estimates are equally credible. Because of the computational load, necessary full uncertainty estimates are not yet available, and efforts to obtain them are ongoing (e.g., Kalmikov & Heimbach 2014). Surely the future lies that way.

7.2. Formal Optimization Methods

Methods exist for optimizing observational strategies subject to practical constraints, if agreement could be reached on specific goals of climate research. Such methods include deterministic and Monte Carlo approaches, as in simulated annealing, genetic algorithms, and the like (e.g., Barth 1992, Hernandez et al. 1995). Apart from the almost intractable issues of agreement on acceptable, specific long-term goals and the huge dimension of the problem, oceanic measurement techniques are not fungible in any reasonable time span: An inference that a goal is best met by deploying 800 moorings does not lead to conversion of satellite, float, or ship resources to that end. Nonetheless, understanding what an optimal strategy might be does provide a critical benchmark for evaluating a practical future proposal.

Among the type of questions that must be posed is whether 20 tide gauges are an adequate substitute for global altimetric measurements in determining sea level change. Exaggeration of the accuracies of estimates from past data coverage can have the unintended consequence of undermining arguments for capable future systems.

7.3. New Technologies

History shows that envisioning effective new technologies more than approximately 20 years in advance is impossible. Existing systems had gestation periods of similar or longer duration (Argo arose from the PALACE floats of WOCE, which in turn were based on the float technology from the 1950s; the notion of accurate altimetry was discussed shortly after the 1957 Sputnik launch and led to the TOPEX/POSEIDON and Jason satellites in 1992). What can or should be anticipated over the next 10–20 years that would be applicable to global observations? Visionaries are needed.

Floats that carry suites of sensors to profile the full water column are a primary candidate. The technology exists and is not cheap, and problems of calibration are known, but no fundamental obstacle has appeared, and exploratory deployments are already under way. Self-powered gliders could become much cheaper and more abundant. High-accuracy altimetry of the TOPEX/POSEIDON and Jason class should evolve into much denser coverage from new swath-type instruments (Fu & Ubelmann 2014).

Whether useful ocean measurements from space extending beyond what we already know [color, sea surface temperature and salinity, surface height, roughness (scatterometry), and gravity/bottom pressure] will emerge is unclear. Are there radically different space-borne technologies needing investment that would let researchers penetrate the water column in the same way that GRACE (Gravity Recovery and Climate Experiment) provides bottom pressure estimates?

The use of natural background acoustic sources (e.g., earthquakes, ship noise, ice breakup, and animals) may yet provide large-scale measurements far more cheaply than do synthetic sound sources. If synthetic or natural sources become a reality at frequencies as low as 1 Hz, ocean acoustic holography could become practical.

Animal-based instrumentation (Roquet et al. 2013) is also likely to continue to be developed. The ultimate limits of this approach (e.g., using deep-diving animals) are not yet known.

8. A SUMMARY AND WHAT IS NEEDED

The global ocean observing problem is fundamentally one of obtaining useful accuracies in a system near equilibrium in the presence of large background values, both of reservoir size and of input/output fields. Apart from temperatures in the upper quarter of the ocean over the last ten years, available determinations of changes in heat content (mean temperature), freshwater content (salinity), or mean sea level are dependent on various untested statistical models. Many results are plausible and possibly even correct, but they are fragile because data are lacking to test the assumptions. A recent inference (Durack et al. 2014, Cheng & Zhu 2015) that oceanic heat uptake over the last few decades has been significantly underestimated is consistent with the general warm bias seen in **Figure 7**, but a reliable multidecadal value is not available. A similar and even greater problem lies with the salinity change calculation. In the combined near-global altimeter-Argo era, changes in hydrographic parameters can be accurately estimated for the region above a depth of approximately 1–2 km, with heat content errors resulting from the missing abyssal data of approximately 10%. As durations increase, the role of the deep ocean can only grow, but it is difficult to quantify without observations. Closing the budget of altimetric mean sea level incurs the same errors in the hydrography in addition to those implicit in the altimetric observations themselves.

Because coupled, temporally and spatially integrating turbulent systems (the ocean and atmosphere, plus cryosphere, biosphere, and land processes) are expected to undergo slow, large-scale exchanges of properties such as heat, they must exhibit what appear to be long-term trends. Externally imposed secular trends (e.g., from greenhouse gas emissions or solar variability) thus cannot be deduced with any confidence without the provision of strong a priori physics. At the present time, none of the externally imposed sources or sinks for heat, freshwater, or energy can be estimated quantitatively with useful confidence several decades in the past (for heat, accuracies better than 0.1 W/m^2 are required). In some cases, values even in the most recent decade remain too uncertain to usefully constrain oceanic trends.

Many questions remain. What data distributions in time, depth, latitude, longitude, and season are required? If those requirements cannot be met, should less useful measurements nonetheless be started? How long do measurements need to be maintained? Should investments be made in new technologies that may ultimately make it possible to meet the requirements? To what extent can securely known physics (e.g., "the signal is globally homogeneous after 30 years") guide the quantitative design?

Uncertainty is a two-edged sword. An inference that changes and trends are poorly known cannot be used to conclude that constructive actions are unwarranted; underestimation is as likely as overestimation. No substitute exists for an adequate observing system, and the solution to these uncertainty problems is unambiguous: extension of quantified observing systems to the entire, top-to-bottom, global ocean, and their sustenance into the indefinite future. This challenge is an intergenerational one. Failure to make adequate measurements leads to both irrecoverable scientific loss and the waste of substantial resources.

That which is far off, and exceeding deep, who can find it out?
—Ecclesiastes 7:24 (King James Version)

SUMMARY POINTS

1. Oceanic reservoir values of scientifically and societally important properties (heat, freshwater, etc.) are very large compared with inferred changes.
2. Most of these properties are in near equilibrium with the atmosphere, cryosphere, and land processes.
3. Consequently, determination of observed change must be done with accuracies and precisions that are challenging under the best of circumstances.
4. Space-time inhomogeneities in both signal and noise in temperature, salinity, and sea surface elevation render estimates prior to the 1990s and 2000s subject to major issues of sampling bias.
5. Upper-ocean heat content changes in the altimeter-Argo era beginning around 2005 have likely been determined with sufficient accuracy to discuss anthropogenic warming of the 25–50% lying in the upper ocean. Simultaneous deep-ocean changes are much more poorly known but probably introduce an error of no more than approximately 10%. As the duration extends beyond a decade or two, physics dictates that deep-ocean content changes must become comparable to those in the upper depths.
6. The compelling scientific problem is to determine the nature of an adequate global observing system for detecting with adequate accuracy and precision oceanic changes in heat, freshwater, sea level change partitioning, and many other properties, such as carbon and oxygen, so that future generations will not be frustrated in their ability to quantify the changing climate system.
7. Combined dynamical/data-determined state estimates are the essential tools for assuring that all relevant data are properly included with uncertainty estimates. The central need is for useful uncertainty estimates for the global fields, a problem of computational complexity and cost.

FUTURE ISSUES

1. In addition to temperature, salinity, and sea level, the central observational fields for oceanic change need to be determined.
2. Quantified practical observation systems are needed to ensure that future scientists will be able to detect global average and integral changes at useful levels.
3. Observation systems will need to be deployed, maintained, and calibrated for the decades required to document changes and their physics (and chemistry and biology) where appropriate.

DISCLOSURE STATEMENT

The author is not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

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